but not sp-open set in X .Thus, f is a s$b-$continuous function, but not soft pre-continuous function.

Theorem 2.13.11 Every soft continuous function is s$b-$continuous function.

Proof Let f : $X \rightarrow$ Y be a soft continuous function. Let (F,K) be a soft open set in Y. Since f is soft continuous, $f^{-1}((F,K))$ is soft open in X .And so $f^{-1}((F,K))$ is s $b-$open set in X .Therefore, f is s$b-$continuous function.

Definition 2.13.12 A mapping $f: X \rightarrow$ Y is said to be soft $b-$irresolute (briefly s $b-$irresolute) if $f^{-1}((F,K))$ is s$b-$closed set in X, for every s$b-$closed set (F,K) in Y.

Theorem 2.13.13 A mapping f : $X \rightarrow$ Y is s$b-$irresolute mapping if and only if the inverse image of every s$b-$open set in Y is s$b-$open set in X .

Theorem 2.13.14 Every s$b-$irresolute mapping is s$b-$continuous mapping.

Proof Let f: X$\rightarrow$Y is s$b-$irresolute mapping. Let (F,K) be a soft closed set in Y, then (F,K) is s$b-$closed set in Y. Since f is s$b-$irresolute mapping, $f^{-1}((F,K))$ is a $sb-$closed set in X .Hence, f is s$b-$continuous mapping.

Theorem 2.13.15 Let $f: (X,E,\tau) \rightarrow (Y,K,\upsilon)$, $g: (Y,K,\upsilon) \rightarrow (Z,T,\lambda)$ be two functions. Then

(i) $g \circ f : X \rightarrow$Z is s$b-$continuous, if f is s$b-$continuous and g is soft continuous.

(ii) $g \circ f : X \rightarrow$ Z is s$b-$irresolute, if f and g are s$b-$irresolute functions.

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(iii) $g \circ f : X \to$ Z is s$b-$continuous if f is s$b-$irresolute and g is s$b-$continuous.

Proof

(i) Let (H,T) be soft closed set of Z. Since g : Y $\to$ Z is soft continuous, by definition $g^{-1}((H,T))$ is soft closed set of Y. Now f : $X \to$ Y is s$b-$continuous and $g^{-1}((H,T))$ is soft closed set of Y, so by definition 2.13.4, $f^{-1}(g^{-1}((H,T))) = (g \circ f)^{-1}((H,T))$ is s$b-$closed in X .Hence $g \circ f : X \to$Z is s $b-$continuous.

(ii) Let g : Y $\to$ Z is s$b-$irresolute and let (H,T) be s$b-$closed set of Z. Since g is s $b-$irresolute by

definition 2.13.12 , $g^{-1}((H,T))$ is s$b-$closed set of Y. Also f : $X \to$Y is s$b-$irresolute, so $f^{-1}(g^{-1}((H,T))) = (g \circ f)^{-1}((H,T))$ is s$b-$closed. Thus, $g \circ f : X \to$ Z is s$b-$irresolute.

(iii)Let (H,T) be soft closed set of Z. Since g: Y $\to$Z is s$b-$continuous, then $g^{-1}((H,T))$ is s$b-$closed set of Y. Also $f : X \to$ Y is s$b-$irresolute, so every s $b-$closed set of Y is s$b-$closed in X under f^{-1}. Therefore, $f^{-1}(g^{-1}((H,T))) = (g \circ f)^{-1}((H,T))$ is s$b-$closed set of X .Thus, $g \circ f : X \to$Z is s$b-$continuous.

Theorem 2.13.16 4.18) Let $f : (X,E,\tau) \to (Y,K,\upsilon)$, $g : (Y,K,\upsilon) \to (Z,T,\lambda)$ be two functions. Then, $g \circ f : X \to$ Z is s $b-$continuous if f is s $b-$irresolute and g is soft α -continuous.

Proof: Let (H,T) be soft closed set of Z. Since g: Y $\to$Z is s$b-$continuous, then $g^{-1}((H,T))$ is s$\alpha-$closed set of Y and hence $g^{-1}((H,T))$ is s$b-$closed set of Y.

Also $f: X \to Y$ is s$b-$irresolute, so every s$b-$closed set of Y is s$b-$closed in X under f^{-1}. Therefore,

$f^{-1}(g^{-1}((H,T))) = (g \circ f)^{-1}((H,T))$ is s$b-$closed set of X .Thus, $g \circ f: X \to Z$ is s$b-$continuous.

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Definition 2.13.17 A mapping $f: X \to Y$ is said to be soft $b-$open (briefly $sb-$open) map if the image of every soft open set in X is $sb-$open set in Y:

Definition 2.13.18 A mapping $f: X \to Y$ is said to be soft $b-$closed (briefly $sb-$closed) map if the image of every soft closed set in X is $sb-$closed set in Y.

Theorem 2.13.19 If $f: X \to Y$ is soft closed function and $g: Y \to Z$ is $sb-$closed function, then $g \circ f$ is $sb-$closed function.

Proof For a soft closed set (F,A) in X , $f((F,A))$ is soft closed set in Y. Since g: $Y \to Z$ is $sb-$closed function, $g(f((F,A)))$ is $sb-$closed set in Z. $g(f((F,A))) = (g \circ f)((F,A)))$ is $sb-$closed set in Z. Therefore, $g \circ f$ is $sb-$closed function.

Theorem 2.13.20 A map $f: X \to Y$ is $sb-$closed if and only if for each soft set (F,K) of Y and for each soft open set (F,A) of X such that $f^{-1}((F,K)) \tilde{\subseteq} (F,A)$, there is a $sb-$open set (G,K) of Y such $(F,K) \tilde{\subseteq} (G,K)$ and $f^{-1}((G,K)) \tilde{\subseteq} (F,A)$

Proof: Suppose f is $sb-$closed map. Let (F,K) be a soft set of Y, and (F,A) be a soft open set of X , such that $f^{-1}((F,K)) \tilde{\subseteq} (F,A)$. Then $(G,K) = (f((F,A)^c))^c$ is a $sb-$open set in Y such $(F,K) \tilde{\subseteq} (G,K)$ and $f^{-1}((G,K)) \tilde{\subseteq} (F,A)$.

Conversely, suppose that (F,B) is a soft closed set of X. Then $f^{-1}((f((F,B)))^c) \tilde{\subseteq} (F,B)^c$, and $(F,B)^c$ is soft open set. By hypothesis, there is a $sb-$open set (G,K) of Y such $(f(F,A))^c \tilde{\subseteq} (G,K)$ and $f^{-1}(G,K) \tilde{\subseteq} (F,B)^c$. Thus $(F,B) \tilde{\subseteq} (f^{-1}(G,K))^c$. Hence $(G,K)^c \tilde{\subseteq} f((F,B)) \tilde{\subseteq} f(f^{-1}((G,K))^c) \tilde{\subseteq} (G,K)^c$, which implies $f((F,B)) = (G,K)^c$. Since $(G,K)^c$ is $sb-$closed set, $f((F,B))$ is s $b-$closed set. So f is a $sb-$closed map.

Theorem 2.13.21 Let f : $X \to$ Y, g : Y $\to$ Z be two maps such that $g \circ f :$ X$\to$Z is $sb-$closed map.

(i) If f is soft continuous and surjective, then g is $sb-$closed map.

(ii) If g is $sb-$irresolute and injective, then f is $sb-$closed map.

Proof

(i) Let (H,K) be a soft closed set of Y. Then, $f^{-1}((H,K))$ is soft closed set in X as f is soft continuous. Since $g \circ f$ is $sb-$closed map $(g \circ f)(f^{-1}((H,K))) = g((H,K))$ is $sb-$closed set in Z. Hence g : Y $\to$ Z is $sb-$closed map.

(ii) Let (H,E) be a soft closed set in X. Then $(g \circ f)((H,E))$ is $sb-$closed set in Z, and so $g^{-1}(g \circ f)((H,E)) = f((H,E))$ is $sb-$closed set in Y. Since g is $sb-$irresolute and injective. Hence f is a s $b-$closed map.

Theorem 2.13.22 If (F,B) is s $b-$closed set in X and $f : X \to$Y is bijective, soft continuous and s $b-$closed, then $f((F,B))$ is s $b-$closed set in Y.

Proof: Let $f((F,B)) \tilde{\subseteq} (F,K)$ where (F,K) is a soft open set in Y. Since f is soft continuous, $f^{-1}((F,K))$ is a soft open set containing (F,B). Hence containing $bcl^S((F,B)) \tilde{\subseteq} f^{-1}((F,K))$ as (F,B) is $sb-$closed set. Since f is $sb-$closed, $f(bcl^S((F,B)))$ is $sb-$closed set contained in the soft open set (F,K), which implies $bcl^S(f(bcl^S((F,B)))) \tilde{\subseteq} (F,K)$ and hence $bcl^S(f(\ ((F,B))) \tilde{\subseteq} (F,K)$. So $f((F,B))$ is $sb-$closed set in Y.

3. SOFT π GR-CLOSED SETS.

In this section, we first introduce the following definitions.

Definition:3.1

A subset (A,E) of a soft topological space X is called

(i) a soft rg -closed set if $cl^S\ (A,E)\ \tilde{\subseteq}\ (U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

(ii) a soft π^* g-closed if $cl^S(\text{int}^S\ (A,E))\ \tilde{\subseteq}\ (U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

(iii) a soft π gα-closed if $\alpha cl^S\ (A,E)\ \tilde{\subseteq}\ (U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

(iv) a soft π gp-closed if $pcl^S\ (A,E)\ \tilde{\subseteq}(U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

(v) a soft π g $b-$closed if $bcl^S\ (A,E)\ \tilde{\subseteq}\ (U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

(vi) a soft π gs-closed if $scl^S\ (A,E)\ \tilde{\subseteq}\ (U,E)$ whenever $(A,E)\ \tilde{\subseteq}\ (U,E)$ and (U,E) is soft π - open.

Definition:3.2

A soft subset (A,E) of a soft topological space X is called a soft π gr-closed set in X if $rcl^S\ (A,E)\ \tilde{\subseteq}(U,E)$ whenever $(A,E)\ \tilde{\subseteq}(U,E)$, where (U,E) is soft π - open in X .The family of all soft π gr -closed sets of X is denoted by S π GRC(X) .

Result :3.3

Every soft regular closed set is soft π gr-closed but not conversely.

Example:3.4

Let $X=\{a,b,c,d\}, E=\{e_1,e_2\}$. Let $F_1,F_2,...,F_6$ are functions from E to *P(X)* and are defined as follows:

$F_1(e_1)=\{c\}, F_1(e_2)=\{a\}$,

$F_2(e_1)=\{d\}, F_2(e_2)=\{b\}$,

$F_3(e_1)=\{c,d\}, F_3(e_2)=\{a,b\}$,

$F_4(e_1)=\{a,d\}, F_4(e_2)=\{b,d\}$

$F_5(e_1)=\{b,c,d\}, F_5(e_2)=\{a,b,c\}$

$F_6(e_1)=\{a,c,d\}, F_6(e_2)=\{a,b,d\}$.

Then $\tau=\{\Phi,(X,E),(F_1,E),(F_2,E),...,(F_6,E)\}$ is a soft topology and elements in τ are soft open sets. The soft closed sets are its relative complements. Here the soft set $(H,E)=\{\{b,c,d\},\{a,b,c\}\}$ is soft π gr-closed but not soft regular closed.

Remark:3.5

The concept of soft closed and soft π gr-closed are independent.

Example:3.6

In Example 3.4, (i) the soft set $(A,E)=\{\{a\},\{d\}\}$ of a soft topological space X is soft closed but not soft π gr-closed in X.

(ii)the soft subset $(H,E)=\{\{b,c,d\},\{a,b,c\}\}$ is soft π gr-closed but not soft closed in X.

Remark: 3.7

The concept of soft g-closed and soft π gr-closed are independent.

Example:3.8

In Example 3.4, (i) the soft set $(A,E)=\{\{a\},\{d\}\}$ of a soft topological space X is soft g-closed but not soft π gr-closed in X.

(ii) the soft subset $(H,E)=\{\{b,c,d\},\{a,b,c\}\}$ is soft π gr-closed but not soft g-closed in X.

Theorem : 3.9

Every soft π gr-closed set is soft π gα-closed, soft π gp-closed, soft π g $b-$closed, soft π gs-closed, soft π g-closed and soft π^* g-closed but not conversely.

Proof: Straight forward.

Example : 3.10

In example 3.4, i)The soft set (A, E) $=\{\{a\},\{d\}\}$ of a soft topological space X is soft $\pi g\alpha$ -closed and soft π g-closed but not soft π gr-closed.

ii)The soft set $(F, E)=\{\{a\},\{b\}\}$ of a soft topological space X is soft π g $b-$closed, soft π gp-closed and soft π gs-closed but not soft π gr-closed.

iii) The soft subset $(G, E)=\{\{c\},\{d\}\}$of topological space X is soft π^* g-closed but not soft π gr-closed.

Theorem :3.11

Every soft π gr-closed set is soft rg-closed.

Proof: Straight forward

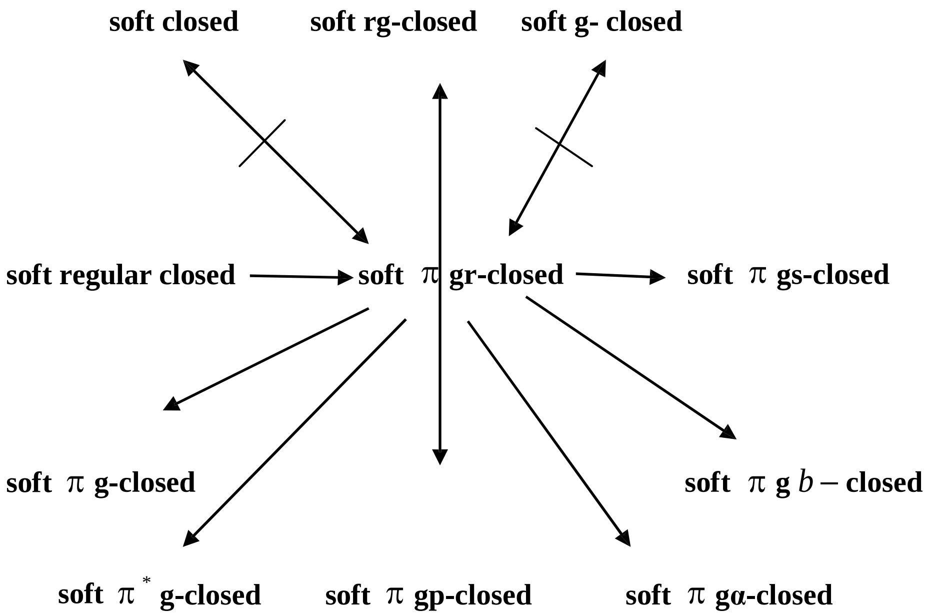
The converse of the above theorem is not true as we see the following example.

Example:3.12

In example 3.4, the soft subset (A, E) $=\{\{a\},\{d\}\}$ of a soft topological space X is soft rg-closed but not soft π gr-closed in X .

Remark:3.13

The relationship between soft π gr-closed sets and soft sets are represented diagrammatically as follows:



Remark:3.14

The union of two soft π gr-closed sets is again a soft π gr-closed set.

Remark:3.15

The intersection of two soft π gr-closed sets need not be soft π gr-closed and is shown in the following example.

Example:3.16

In example 3.4, The soft sets $(I,E)=\{\{b,\mathrm{d}\},\{b,\mathrm{d}\}\}$ and $(J,E)=\{\{a,\mathrm{c},d\},\{a,b,\mathrm{c}\}\}$ are soft π gr-closed sets in X but their intersection $(K,E)=\{\{d\},\{b\}\}$ is not soft π gr-closed in X.

Theorem:3.17

If (A,E) is soft π -open and soft π gr-closed, then it is soft regular closed.

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Proof: Suppose (A, E) is soft π-open and soft π gr-closed. Then $rcl^S(A, E) \,\tilde{\subseteq}\, (A, E)$. But $(A, E) \,\tilde{\subseteq}\, rcl^S(A, E)$. Hence $rcl^S(A, E) = (A, E)$. The above implies (A, E) is soft regular closed .

Result :3.18

If (A, E) is soft π-open and soft π gr-closed, then it is soft closed.

Proof: By (Theorem 3.17) we have (A, E) is soft regular closed and hence soft closed in X .

Theorem:3.19

If a soft subset (A, E) of a soft topological space X is soft π gr-closed set and $(A, E) \,\tilde{\subseteq}\, (B, E) \,\tilde{\subseteq}\, rcl^S(A, E)$. Then (B, E) is also soft π gr-closed subset of X .

Proof: Let (A, E) be a soft π gr-closed set in X and $(B, E) \,\tilde{\subseteq}\, (U, E)$, where (U, E) is soft π-open. Since $(A, E) \,\tilde{\subseteq}\, (B, E)$, $(A, E) \,\tilde{\subseteq}\, (U, E)$. Since (A, E) is soft π gr-closed, thus $rcl^S(A, E) \,\tilde{\subseteq}\, (U, E)$. Given $(B, E) \,\tilde{\subseteq}\, rcl^S(A, E)$. Then $rcl^S(B, E) \,\tilde{\subseteq}\, rcl^S(A, E) \,\tilde{\subseteq}\, (U, E)$. Hence $rcl^S(B, E) \,\tilde{\subseteq}\, (U, E)$ and hence (B, E) is soft π gr-closed.

Theorem:3.20

If (A, E) is soft π gr-closed , then $rcl^S((A, E)) - (A, E)$ does not contain any non-empty soft π-closed set.

Proof: Let (F, E) be a non-empty soft π-closed set such that $(F, E) \,\tilde{\subseteq}\, rcl^S(A, E) - (A, E)$. The above implies $(F, E) \,\tilde{\subseteq}\, X - (A, E)$. Since (A, E) is soft π gr-closed , $X - (A, E)$ is soft π gr-open. Since (F, E) is soft π-closed set, $X - (F, E)$ is soft π-open. Since $rcl^S(A, E) \,\tilde{\subseteq}\, X - (F, E)$, $(F, E) \,\tilde{\subseteq}\, X - rcl^S(A, E)$. Thus

$(F,E)\ \tilde{\subseteq}\ \Phi$, which is a contradiction. The above implies $(F,E)=\Phi$ and hence $rcl^S((A,E))-(A,E)$ does not contain non-empty soft π-closed set.

Corollary:3.21

Let (A,E) be a soft π gr-closed set. Then (A,E) is soft regular closed iff $rcl^S((A,E))-(A,E)$ is soft π-closed.

Proof : Let (A,E) be soft regular closed . Then $rcl^S(A,E)=(A,E)$ and $rcl^S((A,E))-(A,E)=\Phi$,which is soft π-closed. On the other hand, let us suppose that $rcl^S((A,E))-(A,E)$ is soft π-closed. Then by theorem 3.20, $rcl^S((A,E))-(A,E)=\Phi$. The above implies $rcl^S(A,E)=(A,E)$. Hence (A,E) is soft regular closed.

3. 22 SOFT π GR-OPEN SETS.

Let us introduce the following definitions.

Definition: 3.22.1

A soft set (A,E) is called a soft π gr- open set in a soft topological space (X,E,τ) if the relative complement $(A,E)^c$ is soft π gr-closed in (X,E,τ) and the family of all soft π gr –open sets in a soft topological space (X,E,τ) is denoted by S π GRO(X)

Remark: 3.22.2

For a soft subset (A,E) of (X,E), $rcl^S((X,E)-(A,E))=(X,E)-r\operatorname{int}^S(A,E)$.

Theorem: 3.22.3

The soft subset (A,E) of (X,E) is soft π gr-open iff $(F,E)\ \tilde{\subseteq}\ r\operatorname{int}^S(A,E)$ whenever (A,E) is soft π-closed and $(F,E)\ \tilde{\subseteq}\ (A,E)$.

Proof: Let (F,E) be soft π gr-open. Let (F,E) be soft π-closed set and $(F,E)\ \tilde{\subseteq}\ (A,E)$. Then $(X,E)-(A,E)\ \tilde{\subseteq}\ (X,E)-(F,E)$. where $(X,E)-(F,E)$ is soft π

-open. Since (A,E) is soft π gr-open, $(X,E)-(A,E)$ is soft π gr-closed. Then $rcl^S((X,E)-(A,E)) \tilde{\subseteq} (X,E)-(F,E)$. Since $rcl^S((X,E)-(A,E)) = (X,E)-r\text{int}^S(A,E) \Rightarrow (X,E)-r\text{int}^S(A,E) \tilde{\subseteq} (X,E)-(F,E)$. Hence $(F,E) \tilde{\subseteq} r\text{int}^S(A,E)$. On the other hand, let (F,E) be soft π -closed and $(F,E) \tilde{\subseteq} (A,E)$ implies $(F,E) \tilde{\subseteq} r\text{int}^S(A,E)$. Let $(X,E)-(A,E) \tilde{\subseteq} (U,E)$, where $(X,E)-(U,E)$ is soft π -closed. By hypothesis, $(X,E)-(U,E) \tilde{\subseteq} r\text{int}^S(A,E)$. Hence $(X,E)\text{-}r\text{int}^S(A,E) \tilde{\subseteq} (U,E)$. since $rcl^S((X,E)-(A,E)) = (X,E)-r\text{int}^S(A,E)$.The above implies $rcl^S((X,E)-(A,E)) \tilde{\subseteq} (U,E)$,whenever $(X,E)-(A,E)$ is soft π -open. Hence $(X,E)\text{-}(A,E)$ is soft π gr-open in (X,E).

Theorem: 3. 22.4

If $r\text{int}^S(A,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} (A,E)$, and (A,E) is soft π gr-open, then (B,E) is soft π gr-open.

Proof: Given $r\text{int}^S(A,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} (A,E)$. Then $(X,E)-(A,E) \tilde{\subseteq} (X,E)-(B,E) \tilde{\subseteq} rcl^S((X,E)-(A,E))$. Since (A,E) is soft π gr-open, $(X,E)-(A,E)$ is soft π gr-closed. Then $(X,E)-(B,E)$ is also soft π gr-closed. Hence (B,E) is soft π gr-open.

Remark: 3. 22.5

For any soft set (A,E), $r\text{int}^S(rcl^S((A,E))-(A,E)) = \Phi$.

Theorem: 3. 22.6

If $(A,E) \tilde{\subseteq} (X,E)$ is soft π gr-closed, then $rcl^S(A,E)-(A,E)$ is soft π gr-open.

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Proof: Let (A,E) be soft π gr-closed and let (F,E) be a soft π -closed set such that (F,E) $\tilde{\subseteq}$ $rcl^S((A,E))-(A,E)$. Then $(F,E)=\Phi$. So, (F,E) $\tilde{\subseteq}$ $r\operatorname{int}^S(rcl^S((A,E))-(A,E))$. Hence $rcl^S((A,E))-(A,E)$ is soft π gr-open.

Theorem: 3. 22.7

The intersection of two soft π gr- open sets is again a soft π gr-open set.

Proof: Straight forward.

Remark: 3. 22.8

The union of two soft π gr-open sets need not be a soft π gr-open set and is shown in the following example.

Example: 3. 22.9

Let (B,E) ={{ a, c},{ a, c}} and (C,E) ={ {b},{d}} are two soft π gr-open sets. Then their union (D,E) ={{ a, b, c},{ a, c , d }} is not soft π gr-open in (X,E,τ).

3.23. SOFT π GR-$T_{1/2}$ -SPACES.

Let us introduce the following definitions.

Definition: 3.23.1

A soft topological space (X,E,τ) is a soft π gr-$T_{1/2}$ -space if every soft π gr-closed set is soft regular closed.

Theorem: 3.23.2

For a soft topological space (X,E,τ), the following conditions are equivalent.

(i) The soft topological space (X,E,τ) is soft π gr- $T_{1/2}$ -space.

(ii) Every singleton of (X,E) is either soft π -closed or soft regular open.

Proof:

(i) $\Rightarrow$(ii): Let (A,E) be a soft singleton set in (X,E) and let us assume that (A,E) is not soft π-closed. Then (X,E)-(A,E) is not soft π-open and hence (X,E)-(A,E) is trivially soft π gr-closed. Since in a soft π gr- $T_{1/2}$-space, every soft π gr-closed set is soft regular closed. Then (X,E)-(A,E) is soft regular closed. Hence (A,E) is soft regular open.
(ii) $\Rightarrow$(i) : Assume that every singleton of a soft topological space (X,E) is either soft π-closed or soft regular open. Let (A,E) be a soft π gr-closed set in (X,E) . Obviously, (A,E) $\tilde{\subseteq} rcl^S$ (A,E). To prove rcl^S (A,E) $\tilde{\subseteq}(A,E)$, let $(F,E)\tilde{\in}$ $rcl^S(A,E)$, where (F,E) is singleton set, we want to show $(F,E)\tilde{\in}(A,E)$. Now, we have two cases (since (F,E) is either soft π-closed or soft regular open).

Case (i): when (F,E) is soft π-closed, suppose (F,E) does not belong to (A,E). Then $(F,E)\tilde{\subseteq}$ rcl^S $((A,E))$-(A,E), which is a contradiction to the fact that rcl^S ((A,E)) -(A,E) does not contain any non-empty soft π-closed set. Therefore, $(F,E)\tilde{\in}(A,E)$. Hence rcl^S (A,E) $\tilde{\subseteq}(A,E)$. Then (A,E) is soft regular closed and hence every (A,E) soft π gr-closed set is soft regular closed. Hence the soft topological space (X,E) is soft π gr- $T_{1/2}$-space.
Case(ii): when (F,E) is soft regular open in (X,E), we have $(F,E) \sqcap (A,E) \neq \Phi$. (since $(F,E) \in rcl^S$ (A,E)). Hence $(F,E)\tilde{\in}(A,E)$. Therefore, rcl^S $(A,E)\tilde{\subseteq}(A,E)$. Then rcl^S (A,E) =(A,E), thus (A,E) is soft regular closed and hence (X,E) is soft π gr- $T_{1/2}$-space.

Theorem: 3.23.3
(i) $SRO(X,E,\tau)$ $\tilde{\subseteq}$ $S\pi\, RGO(X,E,\tau)$
(ii) A soft topological space (X,E,τ) is π gr- $T_{1/2}$-space iff $SRO(X,E,\tau)= S\pi\, GRO$ (X,E,τ)
Proof:
(i) Let (A,E) be soft regular open. Then $(X,E)-(A,E)$ is soft regular closed and so soft π gr-closed. Hence (A,E) is soft π gr-open and hence $SRO(X,E,\tau)$ $\tilde{\subseteq}$ $S\pi\, GRO$ (X,E,τ)
(ii) Necessity: Let (X,E,τ) be π gr- $T_{1/2}$-space. Let $(A,E) \in$ $S\pi\, GRO(X,E,\tau)$. Then $(X,E)-(A,E)$ is soft π gr-closed. Since the space soft π gr- $T_{1/2}$-space, $(X,E)-(A,E)$ is soft regular closed. The above implies (A,E) is soft regular open in X . Hence $S\pi\, GRO(X,E,\tau)$=$SRO(X,E,\tau)$.

Sufficiency: Let $S\pi GRO(X,E,\tau)=SRO(X,E,\tau)$. Let (A,E) be soft π gr-closed. Then $(X,E)-(A,E)$ is soft π gr-open. Thus $(X,E)-(A,E)\in SRO(X,E,\tau)$ and hence (A,E) is soft regular closed.

EXERCISES

4.1) Let (F,A), (G, B), and (H, C) be three soft sets over U, then

(i) $(F,A) \coprod$ ((G, B) $\coprod$ (H, C)) = ((F,A) ($\coprod$ (G, B)) $\coprod$ (H, G),

(ii) $(F,A) \prod$ ((G, B) $\prod$ (H, G)) = ((F,A) $\prod$ (G, B)) $\prod$ (H, G),

(iii) $(F,A) \coprod$ ((G, B) $\prod$ (H, C)) = ((F,A) $\coprod$ (G, B)) $\prod$ ($(F,A) \coprod$ (H, C)),

(iv) $(F,A) \prod$ ((G, B) $\coprod$ (H, C)) = ((F,A) $\prod$ (G, B)) $\coprod$ ($(F,A) \prod$ (H, C)).

4.2) Let X be a soft topological space. Then,

(i) An arbitrary union of s $b-$ open sets is a s $b-$ open set.

(ii) An arbitrary intersection of s $b-$ closed sets is a s $b-$ closed set.

4.3) In a soft topological space X, (F,A) is s $b-$ closed (s $b-$ open) set if and only if $(F,A)=bcl^s(F,A)$ ($(F,A)=b\text{int}^s(F,A)$).

4.4) In a soft topological space X the following relations hold;

(i) $bcl^s(F,A) \coprod bcl^s(G,B) \cong bcl^s((F,A) \coprod (G,B))$,

(ii) $bcl^s((F,A) \prod (G,B)) \cong bcl^s(F,A) \prod bcl^s(G,B)$,

(iii) $b\text{int}^s(F,A) \coprod b\text{int}^s(G,B) \cong b\text{int}^s((F,A) \coprod (G,B))$,

(iv) $b\text{int}^s((F,A) \prod (G,B)) \cong b\text{int}^s(F,A) \prod b\text{int}^s(G,B)$.

4.5) Let (F,A) be a s $b-$ open set in a soft topological space X.

(i) If (F,A) is a sr-closed set then (F,A) is a sp-open set.

(ii) If (F,A) is a sr-open set then (F,A) is a ss-open set.

4.6) Let (F, A) (be a s $b-$open set in a soft topological space X.

(i) If (F, A) is a sr-closed set then (F, A) is a ss-closed set.

(ii) If (F, A) is a sr-open set then (F, A) is a sp-closed set.

4.7) For any s $b-$open set (F, A) in soft topological space X, $cl^{s}(F, A)$ is sr-closed set.

4.8) For any s $b-$closed set (F, A) in a soft topological space X, $\operatorname{int}^{s}(F, A)$ is sr-open set.

4.9) Let (F, A) be a s $b-$open (s $b-$closed) set in a soft topological space X, such that $\operatorname{int}^{s}(F, A) = \Phi$. Then, (F, A) is a sp-open set.

4.10) Let (F, A) be a s $b-$open (s $b-$closed) set in a soft topological space X, such that $cl^{s}(F, A) = \Phi$. Then, (F, A) is a ss-open set.

4.11) Let (F, A) be a set of a soft topological space X. Then,

(i) $sbcl((F, A)) \tilde{\subseteq} sscl((F, A)) \prod spcl((F, A))$,

(ii) $ss\operatorname{int}((F, A)) \coprod sp\operatorname{int}((F, A)) \tilde{\subseteq} sb\operatorname{int}((F, A))$.

4.12) Let X be a soft topological space. If (F, A) is an soft open set and (G, B) is a s $b-$open set in X. Then $(F, A) \prod (G, B)$ is a s $b-$open set in X.

4.13) Let X be soft topological space. If (F, A) is an sa-open set and (G, B) is a s $b-$open set in X. Then $(F, A) \prod (G, B)$ is a s $b-$open set in X.

4.14) (F, A) is s $b-$open (s $b-$closed) set in X if and only if (F, A) is the union (intersection) of ss-open set and s $b-$open set in X.

4.15) Every soft continuous function is soft β-continuous function.

4.16) A mapping $f: X \to$ Y is s $b-$irresolute mapping if and only if the inverse image of every soft β-open set in Y is soft $b-$open set in X.

4.17) Every s $b-$irresolute mapping is soft β-continuous mapping.