The implication in (5) is not reversible, in general, as is shown in the following.

Example 2.31. Let $FS(X,E)$, $FS(Y,T)$ be classes of fuzzy soft sets and $f: FS(X,E) \to FS(Y,T)$ as defined in Example 2.22. For (5) define mappings $u: X \to Y$ and $\psi: E \to T$ as

u(a)=y, u(b)=y, u(c)=z,

$\psi(e_1)=t_1$, $\psi(e_2)=t_2$, $\psi(e_3)=t_2$, $\psi(e_4)=t_1$.

Choose two fuzzy soft sets in $FS(Y,T)$ as

$F_A = \{t_3 \setminus (x,0.8),(y,0),(z,0)\}$,

$G_B = \{t_3 \setminus (x,0.3),(y,0.1),(z,0.5)\}$.

Then calculations give

$f^{-1}(F_A) = \Phi \,\breve{\subseteq}\, \Phi = f^{-1}(G_B)$, but $F_A \,\breve{\not\subset}\, G_B$.

3. Fuzzy soft point and its neighborhood structure

Definition 3.1. A fuzzy soft point F_e over (U,E) is a special fuzzy soft set, defined by $F_e(a) = \mu_{F_e}$ if $a = e$; where $\mu_{F_e} \neq \bar{0}$ if $a \neq e$.

Definition 3.2. Let F_A be a fuzzy soft set over (U,E) and G_e be a fuzzy soft point over (U,E). Then we say that $G_e \in F_A$ if and only if $\mu_{G_e} \subseteq \mu^e_{F_A} = F_A(e)$ i.e., $\mu_{G_e}(x) \leq \mu^e_{F_A}(x)$ for all $x \in U$.

Definition 3.3. A fuzzy soft set F_A is said to be a neighborhood of a fuzzy soft point G_e if there exists $H_B \in \tau$ such that $G_e \in H_B \breve{\subseteq} F_A$. Then clearly, every open fuzzy soft set is a neighborhood of each of its points.

Theorem 3.4. Let $F_A \in FS(U,E)$. Then $F_A \in \tau$ if and only if F_A is a neighborhood of each of its fuzzy soft points.

Proof. If $F_A \in \tau$, then obviously F_A is a neighborhood of each of its fuzzy soft points. Conversely, let F_A is a neighborhood of each of its fuzzy soft points. Then for any $F_e^\alpha \in F_A, \alpha \in \Gamma$, there exists $G_{A_e^\alpha}^\alpha \in \tau$ such that $F_e^\alpha \in G_{A_e^\alpha}^\alpha \breve{\subseteq} F_A$. So that $\breve{\vee} F_e^\alpha \breve{\subseteq} \breve{\vee} G_{A_e^\alpha}^\alpha \breve{\subseteq} F_A$ _ _ _ (1)[where union is taken over the set of all $\alpha \in \Gamma$ and all $e \in E$].

We now show that $\breve{\vee} F_e^\alpha = F_A$. Since each $F_e^\alpha(a) \breve{\subseteq} F_A(a)$, where $e \in E$ and $\alpha \in \Gamma$, there exists $\alpha \in \Gamma$ such that $F_e^\alpha(a) = F_A(a)$. Therefore $\breve{\vee} F_e^\alpha(a) = F_A(a)$, where union is taken over the set of all $\alpha \in \Gamma$ and all $e \in E$. It implies that $\breve{\vee} F_e^\alpha = F_A$ _ _ _ (2)

From (1) and (2) we get $F_A = \breve{\vee} G_{A_e^\alpha}^\alpha$. Again since each $G_{A_e^\alpha}^\alpha \in \tau, \breve{\vee} G_{A_e^\alpha}^\alpha \in \tau$. Hence $F_A \in \tau$.

Definition 3.5. The collection of all neighborhoods of a point F_e over (U,E) is called the neighborhood system at F_e and it is denoted by η_{F_e}.

Theorem 3.6. The neighborhood system η_{F_e} at any point F_e over (U,E) satisfy the following properties

(i) $\eta_{F_e} \neq \phi$,

(ii) $G_A \in \eta_{F_e} \Rightarrow F_e \in \eta_{F_e}$,

(iii) $G_A, H_B \in \eta_{F_e} \Rightarrow G_A \breve{\wedge} H_B \in \eta_{F_e}$

(iv) $G_A \in \eta_{F_e}$ *and* $G_A \breve{\subseteq} H_B \Rightarrow H_B \in \eta_{F_e}$.

Proof. (i) Since $\overline{E} \in \tau$ and $F_e \in \overline{E}$, thus $\overline{E} \in \eta_{F_e}$ and hence $\eta_{F_e} \neq \phi$

(ii) Obvious.

(iii) Since G_A and $H_B \in \eta_{F_e}$, there exist V_{A_1} and W_{B_1} in τ such that $F_e \in V_{A_1} \breve{\subseteq} G_A$ and $F_e \in W_{B_1} \breve{\subseteq} H_B$. Thus $\mu_{F_e}(x) \leq \mu^e_{V_{A_1}}(x)$ and $\mu_{F_e}(x) \leq \mu^e_{W_{B_1}}(x)$ for all $x \in U$. Therefore we have $\mu_{F_e}(x) = \min\{\mu^e_{V_{A_1}}(x), \mu^e_{W_{B_1}}(x)\}$ for all $x \in U$. So, $\mu_{F_e} \in \mu^e_{V_{A_1}}(x) \cap \mu^e_{W_{B_1}}(x)$ $^e_{W_{B_1}}$. That is, $F_e \in V_{A_1} \breve{\wedge} W_{B_1} \breve{\subseteq} G_A \breve{\wedge} H_B$. Again since $V_{A_1} \breve{\wedge} W_{B_1} \in \tau$, $G_A \breve{\wedge} H_B \in \eta_{F_e}$.

(iv) Obvious.

Definition 3.7. The union of all fuzzy soft open subsets of F_A over (U, E) is called the interior of F_A and is denoted by $\text{int}^{fs}(F_A)$.

Example 3.8. Let $E = \{e_1, e_2, e_3\}$, $U = \{c_1, c_2, c_3\}$ and A, B, C be the subsets of E, where $A = \{e_1, e_2\}$, $B = \{e_2, e_3\}$ and $C = \{e_1, e_3\}$ and also let $\tau = \{\Phi, \overline{E}, F_A, G_B, H_{e_2}, I_E, J_B, K_{e_2}\}$ be a fuzzy soft topology over (U, E) where $F_A, G_B, H_{e_2}, I_E, J_B, K_{e_2}$ are fuzzy soft set over (U, E), defined as follows

$\mu^{e_1}_{F_A} = \{0.5, 0.75, 0.4\}$, $\mu^{e_2}_{F_A} = \{0.3, 0.8, 0.7\}$,

$\mu^{e_2}_{G_B} = \{0.4, 0.6, 0.3\}$, $\mu^{e_3}_{G_B} = \{0.2, 0.4, 0.45\}$,

$\mu_{H_{e_2}} = \{0.3, 0.6, 0.3\}$,

$\mu^{e_1}_{I_E} = \{0.5, 0.75, 0.4\}$, $\mu^{e_2}_{I_E} = \{0.4, 0.8, 0.7\}$, $\mu^{e_3}_{I_E} = \{0.2, 0.4, 0.45\}$

$\mu^{e_2}_{J_B} = \{0.4, 0.8, 0.7\}$, $\mu^{e_3}_{J_B} = \{0.2, 0.4, 0.45\}$,

$\mu_{K_{e_2}} = \{0.3, 0.8, 0.7\}$,

Now let us consider a fuzzy soft set L_E as follows

$\mu_{L_E}^{e_1} = \{0.7, 0.8, 0.5\}$, $\mu_{L_E}^{e_2} = \{0.4, 0.9, 0.7\}$, $\mu_{L_E}^{e_3} = \{0.2, 0.3, 0.1\}$.

Therefore $\text{int}^{fs}(L_E) = F_A \,\tilde{\vee}\, H_{e_2} \,\tilde{\vee}\, K_{e_2} = F_A$

Proposition 3.9. $\text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B) = \text{int}^{fs}(F_A) \,\tilde{\wedge}\, \text{int}^{fs}(G_B)$.

Proof. Since $F_A \,\tilde{\wedge}\, G_B \,\tilde{\subseteq}\, F_A$, thus $\text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B) \,\tilde{\subseteq}\, \text{int}^{fs}(F_A)$. Similarly, $\text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B) \,\tilde{\subseteq}\, \text{int}^{fs}(G_B)$. Therefore $\text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B) \,\tilde{\subseteq}\, \text{int}^{fs}(F_A) \,\tilde{\wedge}\, \text{int}^{fs}(G_B)$. Let $H_C \in \tau$ such that $H_C \,\tilde{\subseteq}\, \text{int}^{fs}(F_A) \,\tilde{\wedge}\, \text{int}^{fs}(G_B)$. Then $H_C \,\tilde{\subseteq}\, \text{int}^{fs}(F_A)$ and $H_C \,\tilde{\subseteq}\, \text{int}^{fs}(G_B)$. That is $H_C(e) \subseteq F_A(e) \cap G_B(e) = (F_A \,\tilde{\wedge}\, G_B)(e)$ for all $e \in E$. So, $H_C(e) \subseteq F_A(e) \cap G_B(e) = (F_A \,\tilde{\wedge}\, G_B)(e)$ for all $e \in E$. Thus $H_C \,\tilde{\subseteq}\, (F_A \,\tilde{\wedge}\, G_B)$. So $H_C = \text{int}^{fs}(H_C) \,\tilde{\subseteq}\, \text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B)$ This implies that $\text{int}^{fs}(F_A) \,\tilde{\wedge}\, \text{int}^{fs}(G_B) \,\tilde{\subseteq}\, \text{int}^{fs}(F_A \,\tilde{\wedge}\, G_B)$ This completes the proof.

Definition 3.10. Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

Example 3.11. Let us consider the example 3.8 and a fuzzy soft set L_{e_2}, where $\mu_{L_{e_2}} = \{0.5, 0.2, 0.6\}$. Then $L_{e_2} \,\tilde{\subseteq}\, G^c_B$. Therefore $cl^{fs}(L_{e_2}) = G_B^c \,\tilde{\wedge}\, H_{e_2}^c = G_B^c$.

4. Fuzzy soft compact spaces

Definition 4.1. Let Ψ be a collection of fuzzy soft sets. Then we say that Ψ is a cover of a fuzzy soft set F_A if $F_A \,\tilde{\subseteq}\, \tilde{\vee}\{G_B : G_B \in \Psi\}$. Further, if each member of

Ψ is a fuzzy soft open set . Then we say that Ψ is a fuzzy soft open cover. Also, if H is a subfamily of Ψ which is also a cover. Then we say H is a subcover of Ψ.

Definition 4.2. Assume that (X,E,τ) is fuzzy soft topological space and $F_A \in FS(X,E)$. Then we say that F_A is a fuzzy soft compact if and only if for each fuzzy soft open cover of F_A has a finite subcover. Moreover, for any fuzzy soft topological space (X,E,τ) is said to be compact if each fuzzy soft open cover of $\overline{E}$ has a finite subcover.

Example 4.3. A fuzzy soft topological space (X,E,τ) is compact if X is finite.

Example 4.4. Let (X,E,τ) and (Y,T,σ) be two fuzzy soft topological spaces and $\tau \,\breve{\subseteq}\, \sigma$. Then, fuzzy soft topological space (X,E,τ) is compact if (Y,T,σ) is compact.

Proposition 4.5. Let G_B be a fuzzy soft closed set in fuzzy soft compact space (X,E,τ). Then G_B is also compact.

Proof. Let $\Gamma=\{ F^i_{A_i} : i\in I \}$ be any open covering of G_B, where I an index set. Then $\overline{E} \,\breve{\subseteq}\, (\breve{\bigvee}_{i\in I} F^i_{A_i}) \,\breve{\vee}\, G_B{}^c$, that is, $F^i_{A_i}$ together with fuzzy soft open set $G_B{}^c$ is a open covering of $\overline{E}$. Therefore there exists a finite subcovering $F^1_{A_1}, F^2_{A_2}, ..., F^n_{A_n}, G_B{}^c$. Hence we obtain $\overline{E} \,\breve{\subseteq}\, F^1_{A_1} \,\breve{\vee}\, F^2_{A_2} \,\breve{\vee} ... \breve{\vee}\, F^n_{A_n} \,\breve{\vee}\, G_B{}^c$. Therefore, we get $G_B \,\breve{\subseteq}\, F^1_{A_1} \,\breve{\vee}\, F^2_{A_2} \,\breve{\vee} ... \breve{\vee}\, F^n_{A_n} \,\breve{\vee}\, G_B{}^c$ which clearly implies $G_B \,\breve{\subseteq}\, F^1_{A_1} \,\breve{\vee}\, F^2_{A_2} \,\breve{\vee} ... \breve{\vee}\, F^n_{A_n}$ since $G_B \,\breve{\wedge}\, G_B{}^c = \Phi$. Hence G_B has a finite subcovering and so is compact.

Definition 4.6. Let (X,E,τ) be a fuzzy soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist fuzzy soft open sets F_A and G_B such that $x \in F_A$, $y \in G_B$ and $F_A \mathrel{\breve{\wedge}} G_B = \Phi$, then (X,E,τ) is called a fuzzy soft Hausdorff space.

Proposition 4.7. Let G_B be a fuzzy soft compact set in fuzzy soft Hausdorff space (X,E,τ). Then G_B is closed.

Proof. Let $x \in G_B{}^c$. For each $y \in G_B$, we have $x \neq y$, so there are disjoint fuzzy soft open sets F_A^y and H_C^y so that $x \in F_A^y$ and $y \in H_C^y$. Then $\{ H_C^y : y \in G_B \}$ is an fuzzy soft open cover of G_B. Let $\{ H_C^{y_1}, H_C^{y_2}, ..., H_C^{y_3} \}$ be a finite subcover. Then $\breve{\bigwedge}_{i=1}^{n} F_A^{y_i}$ is an open set containing x and contained in $G_B{}^c$. Thus $G_B{}^c$ is fuzzy soft open and G_B is closed.

Theorem 4.8. Let (X,E,τ) and (Y,T,σ) be fuzzy soft topological spaces and $(u, \psi) : (X,E,\tau) \to (Y,T,\sigma)$ continuous and onto fuzzy soft function. If (X,E,τ) is fuzzy soft compact, then (Y,T,σ) is fuzzy soft compact,

Proof. To prove that (Y,T,σ) is a fuzzy soft compact, we will use Theorem 2.28. and Theorem 2.30. Let $F_{A_i}^i$ be any open covering of $\overline{T}$, i.e., $\overline{T} \mathrel{\breve{\subseteq}} \breve{\bigvee}_{i \in I} F_{A_i}^i$. Then $(u,\psi)^{-1}(\overline{T}) \mathrel{\breve{\subseteq}} (u,\psi)^{-1}(\breve{\bigvee}_{i \in I} F_{A_i}^i)$ and $(\overline{E}) \mathrel{\breve{\subseteq}} \breve{\bigvee}_{i \in I}((u,\psi)^{-1}(F_{A_i}^i))$. So $(u,\psi)^{-1}(F_{A_i}^i)$ is an open covering of $\overline{E}$. As (X,E,τ) is compact, there are *1, 2, ..., n* in *I* such that $\overline{E} \mathrel{\breve{\subseteq}} \breve{\bigvee}_{i=1}^{n}((u,\psi)^{-1}(F_{A_i}^i))$. Since (ϕ, ψ) is surjective, we have

$\overline{T} = (u,\psi)(\overline{E}) \breve{\subseteq} (u,\psi)(\breve{\bigvee}_{i=1}^{n}(u,\psi)^{-1}(F^{i}_{A_i})) = \breve{\bigvee}_{i=1}^{n}((u,\psi)(u,\psi)^{-1}(F^{i}_{A_i})) = \breve{\bigvee}_{i=1}^{n} F^{i}_{A_i}$. So we have $\overline{T} \breve{\subseteq} \breve{\bigvee}_{i=1}^{n} F^{i}_{A_i}$, i.e., $\overline{T}$ is covered by a finite number of $F^{i}_{A_i}$. Hence (Y,T,σ) is compact.

Definition 4.9. Let (X,E,τ) and (Y,T,σ) be two fuzzy soft topological spaces. A fuzzy soft mapping $(u,\psi):(X,E,\tau)\rightarrow(Y,T,\sigma)$ is called fuzzy soft closed if $(u,\psi)(F_A)$ is fuzzy soft closed set in (Y,T,σ), for all fuzzy soft closed set F_A in (X,E,τ).

Theorem 4.10. Let (X,E,τ) be a fuzzy soft topological space and (Y,T,σ) be a fuzzy soft Hausdorff space. Fuzzy soft mapping (u,ψ) is closed if fuzzy soft mapping $(u,\psi):(X,E,\tau)\rightarrow(Y,T,\sigma)$ is continuous.

Proof. Let G_B be any fuzzy soft closed set in (X,E,τ). By theorem 4.5 we have G_B is compact. Since fuzzy soft mapping (u,ψ) is continuous, fuzzy soft set $(u,\psi)(G_B)$ is compact in (Y,T,σ). As (Y,T,σ) is fuzzy soft Hausdorff space, fuzzy soft set $(u,\psi)(G_B)$ is closed. Then Fuzzy soft mapping (u,ψ) is closed.

Definition 4.11. A family Γ of fuzzy soft sets has the finite intersection property if the intersection of the members of each finite subfamily of Γ is not the null fuzzy soft set.

Theorem 4.12. A fuzzy soft topological space is compact if and only if each family of fuzzy soft closed sets with the finite intersection property has a nonnull intersection.

Proof. Let Γ be any family of fuzzy soft closed subset such that $\breve{\wedge}\{F^{i}_{A_i} : F^{i}_{A_i} \in \Gamma, i \in I\} = \Phi$. Consider $\Omega = \{(F^{i}_{A_i})^{c} : F^{i}_{A_i} \in \Gamma, i \in I\}$. So Ω is a fuzzy soft open cover of $\overline{E}$. As fuzzy soft topological space is compact, there exists a finite subcovering $(F^1_{A_1})^c, (F^2_{A_2})^c, \ldots, (F^n_{A_n})^c$. Then $\overset{n}{\underset{i=1}{\tilde{\wedge}}} F^i_{A_i} = \overline{E - \overset{n}{\underset{i=1}{\tilde{\vee}}} F^i_{A_i}} = \overline{E - E} = \Phi$. Hence Γ can not have finite intersection property.

Conversely, Assume that a fuzzy soft topological space is not compact. Then any fuzzy soft open cover of $\overline{E}$ has not a finite subcover. Let $\{F^i_{A_i} : i \in I\}$ be fuzzy soft open cover of $\overline{E}$. So $\overset{n}{\underset{i=1}{\tilde{\vee}}} F^i_{A_i} \neq \overline{E}$. Therefore $\overset{n}{\underset{i=1}{\tilde{\wedge}}} (F^i_{A_i})^c \neq \Phi$. Thus, $\{(F^i_{A_i})^c : i \in I\}$ have finite intersection property. By using hypothesis, $\underset{i\in I}{\tilde{\wedge}} F^i_{A_i} \neq \Phi$ and we have $\underset{i\in I}{\tilde{\vee}} F^i_{A_i} \neq \overline{E}$. This is a contradiction. Thus the fuzzy soft topological space is compact.

5. Q-neighborhood structure and accumulation point

Definition 5.1. A fuzzy soft point G_e is said to be a quasi-coincident with F_A, denoted by $G_e \, q \, F_A$ if and only if $\mu_{G_e}(x) + \mu^e_{F_A}(x) > 1$ for some $x \in U$.

Definition 5.2. A fuzzy soft set H_A is said to be a quasi-coincident with F_B, denoted by $H_A \, q \, F_B$ if and only if $\mu^e_{H_A}(x) + \mu^e_{F_B}(x) > 1$ for some $x \in U$ and $e \in A \bigcap B$.

Definition 5.3. A fuzzy soft set F_A is called a Q-neighborhood of G_e if and only if there exists $H_B \in \tau$ such that $G_e \, q \, H_B$ and $H_B \,\tilde{\subseteq}\, F_A$.

Proposition 5.4. $H_B \,\tilde{\subseteq}\, F_A$ if and only if H_B and $F_A{}^c$ are not quasi-coincident. In particular, $G_e \in H_B$ if and only if G_e is not a quasi-coincident with $H_B{}^c$.

Proof. This follows from the fact:

$H_B \,\breve{\subseteq}\, F_A \Leftrightarrow \mu^e_{H_B}(x) \le \mu^e_{F_A}(x)$ for all $x \in U$ and $e \in E \Leftrightarrow$ $\mu^e_{H_B}(x) + \mu^e_{F_A^c}(x) = \mu^e_{H_B}(x) + 1 - \mu^e_{F_A}(x) \le 1$ for all $x \in U$ and $e \in E$.

Proposition 5.5. Let ζ_{G_e} be a family of Q-neighborhood of a fuzzy soft point G_e in a fuzzy soft topological space τ.

(i) If $F_A \in \zeta_{G_e}$, then G_e is quasi-coincident with F_A,

(ii) If $F_A \in \zeta_{G_e}$ and $F_A \,\breve{\subseteq}\, H_B$, then $H_B \in \zeta_{G_e}$,

(iii) If $F_A \in \zeta_{G_e}$, then there exists $H_B \in \zeta_{G_e}$ such that $H_B \,\breve{\subseteq}\, F_A$ and $H_B \in \zeta_{I_d}$ for every fuzzy soft point I_d which is quasi-coincident with H_B.

Proof. (i) suppose $F_A \in \zeta_{G_e}$. Then there exists $I_C \in \tau$ such that $G_e \, q \, I_C$ and $I_C \,\breve{\subseteq}\, F_A$. That is, $\mu_{G_e}(x_0) + \mu^e_{I_C}(x_0) > 1$ for some $x_0 \in U$. Again $\mu^e_{I_C}(x) \le \mu^e_{F_A}(x)$ for all $x \in U$. Therefore $\mu_{G_e}(x_0) + \mu^e_{F_A}(x_0) \ge \mu_{G_e}(x_0) + \mu^e_{I_C}(x_0) > 1$. Hence G_e is quasi-coincident with F_A.

(ii) obvious.

(iii) Suppose $F_A \in \zeta_{G_e}$. Then there exists $H_B \in \zeta_{G_e}$ such that $G_e \, q \, H_B$ and $H_B \,\breve{\subseteq}\, F_A$. That is, there exists $H_B \in \zeta_{G_e}$ such that $G_e \, q \, H_B$ and $H_B \,\breve{\subseteq}\, F_A$. Let I_d be any fuzzy soft point which is quasi-coincident with H_B. Therefore $H_B \in \zeta_{I_d}$.

Proposition 5.6. Let $\{F^j_{A_j}\}_{j \in \Gamma}$ be a family of fuzzy soft sets over (U, E). Then a fuzzy soft point G_e is quasi-coincident with $\breve{\vee} F^j_{A_j}$ if and only if $G_e \, q \, F^j_{A_j}$ for some $j \in \Gamma$.

Proof. Obvious.

Theorem 5.7. A subfamily β of a fuzzy soft topology τ over (U,E) is a base for τ if and only if for each fuzzy soft point G_e and for each Q-neighborhood F_A of G_e, there exists a member $H_B \in \beta$ such that $G_e\, q\, H_B$ and $H_B \tilde{\subseteq} F_A$.

Proof. First we suppose that β is a base for τ. Let G_e be a fuzzy soft point and F_A be a Q-neighborhood of G_e. Then there exists $I_C \in \tau$ such that $G_e\, q\, I_C$ and $I_C \tilde{\subseteq} F_A$. Since $I_C \in \tau$ and β is a base for τ, by theorem 2.19, I_C can be expressed as $\tilde{\bigvee}_{j\in J} H_{B_j}$ where $H_{B_j} \in \beta$ for all $j \in J$. Therefore G_e is a quasi-coincident with $\tilde{\bigvee}_{j\in J} H_{B_j}$. So there exists some H_{B_j} such that $G_e\, q\, H_{B_j}$ and $H_{B_j} \tilde{\subseteq} F_A$. This proves the necessary part of the theorem. We shall now prove the sufficient part of the theorem. If possible, let β is not a base for τ. Then there exists $F_A \in \tau$ such that $G = \tilde{\vee}\{H_B \in \beta \mid H_B \tilde{\subseteq} F_A\} \neq F_A$. Therefore there exists $e \in E$ such that $\mu^e_G(x) < \mu^e_{F_A}(x)$ for some $x \in U$. Thus $\mu^e_{F_A}(x) + 1 - \mu^e_G(x) > 1$. That is $I_e\, q\, F_A$ where $\mu_{I_e}(x) = 1 - \mu^e_G(x)$. So by the given condition there exists $H_B \in \beta$ such that $I_e\, q\, H_B$ and $H_B \tilde{\subseteq} F_A$. Since $H_B \in G$, it follows that $\mu^e_{H_B}(x) \leq \mu^e_G(x)$. That is $\mu^e_{H_B}(x) + \mu_{I_e}(x) \leq 1$, which contradicts the fact that $I_e\, q\, H_B$. This completes the proof.

Theorem 5.8. A fuzzy soft point $G_e \in cl^{fs}(F_A)$ if and only if each Q-neighborhood of G_e is a quasi-coincident with F_A.

Proof. $G_e \in cl^{fs}(F_A)$ if and only if for every closed set H_B containing $F_A, G_e \in H_B$ i.e., $\mu^e_{H_B}(x) \geq \mu_{G_e}(x)$ for all $x \in U$. That is, $G_e \in cl^{fs}(F_A)$ if and only if $1 - \mu^e_{H_B}(x) \leq 1 - \mu_{G_e}(x)$ for all $x \in U$ and for all closed set $F_A \tilde{\subseteq} H_B$. Therefore $G_e \in cl^{fs}(F_A)$ if and only if for any fuzzy soft open set $I_C \tilde{\subseteq} F_A^c$ we have

$\mu^{e}_{I_C}(x) \le 1-\mu_{G_e}(x)$ for all $x \in U$. In other words, for every fuzzy soft open set I_C satisfying $\mu^{e}_{I_C}(x) > 1-\mu_{G_e}(x)$ for some $x \in U$, I_C is not contained in $F_A{}^c$. Again I_C is not contained in $F_A{}^c$ if and only if I_C is a quasi-coincident with F_A. We have thus proved that $G_e \in cl^{fs}(F_A)$ if and only if every open Q-neighborhood I_C of G_e is quasi -coincident with F_A, which is evidently equivalent to what we want to prove.

Definition 5.9. A fuzzy soft point G_e is called an adherence point of a fuzzy soft set F_A if and only if every Q-neighborhood of G_e is a quasi-coincident with F_A.

Proposition 5.10. Every fuzzy soft point of F_A is an adherence point of F_A.

Proof. Obvious

Definition 5.11. A fuzzy soft point G_e is called an accumulation point of a fuzzy soft set F_A if G_e is an adherence point of F_A and every Q-neighborhood of G_e and F_A are quasi-coincident at some fuzzy soft point different from *e*, whenever $G_e \in F_A$. The union of all accumulation points of F_A is called the derived set of F_A, denoted by $F_A{}^d$.

Theorem 5.12. $cl^{fs}(F_A) = F_A \breve{\vee} F_A{}^d$

Proof. Let $\rho = \{G_e : G_e$ is an adherent point of $F_A\}$. Then by theorem 5.8, $cl^{fs}(F_A) = \breve{\vee}\rho$. Now $G_e \in \rho$ if and only if either $G_e \in F_A$ or $G_e \in F_A{}^d$. Hence $cl^{fs}(F_A) = \breve{\vee}\rho = F_A \breve{\vee} F_A{}^d$.

Corollary 5.13. A fuzzy soft set $F_A \in FS(U,E)$ is closed in a fuzzy soft topological space (U,E,τ) if and only if F_A contains all its accumulation points.

Proof. Obvious from the theorem 5.12.

EXERCISES

5.1) Let (U,E,ψ) be a fuzzy soft topological space and β be a sub collection of ψ such that every member of ψ is a union of some members of β. Then β is a fuzzy soft base for the fuzzy soft topology ψ on (U,E).

5.2) A collection of fuzzy soft sets over (U,E) is a subbase for a suitable fuzzy soft topology ψ if and only if

(i) $\Phi \in \Omega$ or Φ is the intersection of a finite number of members of .

(ii) $\overline{E} = \tilde{\vee}\Omega$.

5.3) Let $(u, \psi):(X,E,\tau)\rightarrow (Y,T,\sigma)$ be a fuzzy soft continuous closed mapping from fuzzy soft compact space (X,E,τ) on to fuzzy soft space (Y,T,σ). Then $(u, \psi)(G_B)$ is fuzzy soft compact set in (Y,T,σ), if G_B is a fuzzy soft closed set in fuzzy soft compact space (X,E,τ)

5.4) Let $(u, \psi):(X,E,\tau)\rightarrow (Y,T,\sigma)$ be a fuzzy soft continuous from fuzzy soft compact space (X,E,τ) on to fuzzy soft Hausdorff space (Y,T,σ). Then $(u, \psi)(G_B)$ is fuzzy soft compact set in (Y,T,σ), if G_B is a fuzzy soft closed set in fuzzy soft compact space (X,E,τ)

5.5) Let $(u, \psi):(X,E,\tau)\rightarrow (Y,T,\sigma)$ be a fuzzy soft continuous from fuzzy soft compact space (X,E,τ) on to fuzzy soft Hausdorff space (Y,T,σ). Then $(u, \psi)(G_B)$ is fuzzy soft closed set in (Y,T,σ), if G_B is a fuzzy soft closed set in fuzzy soft compact space (X,E,τ)

5.6) Let ζ_{G_e} be a family of Q-neighborhood of a fuzzy soft point G_e in a fuzzy soft topological space τ. If H_B is not quasi-coincident with $F_A \in \zeta_{G_e}$, then $H_B^{\ c} \in \zeta_{G_e}$.

5.7) H_B and F_A are quasi-coincident if and only if $F_A^{\ c}$ doesn't contain H_B.

5.8) Let $\{F_{A_j}^{j}\}_{j\in\Gamma}$ be a family of fuzzy soft sets over (U,E). Then a fuzzy soft point G_e is not quasi-coincident with $\tilde{\vee} F_{A_j}^{j}$ if and only if G_e and $F_{A_j}^{j}$ are not quasi-coincident for all $j \in \Gamma$.

5.8) A subfamily β of a fuzzy soft topology τ over (U,E) is a base for τ if and only if for each fuzzy soft point G_e and for each Q-neighborhood F_A of G_e , there exists a member $H_B \in \beta$, where H_B is quasi-coincident with G_e but not quasi-coincident with F_A^c .

5.9) A fuzzy soft point $G_e \notin cl^{fs}(F_A)$ if and only if there exists Q-neighborhood of G_e is not quasi-coincident with F_A .

5.10) A fuzzy soft point $G_e \notin cl^{fs}(F_A)$ Each Q-neighborhood of G_e is quasi-coincident with F_A if and only if G_e is not quasi-coincident with $(cl^{fs}(F_A))^c$.