Chapter 1: Whole Numbers 1

Our system of numbers, the decimal-number system, uses ten symbols called digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Numbers in the decimal system can have one or more digits. Each digit in a number that contains two or more digits must be arranged in a specific order to have the value we intend for the number to have. One set of numbers in the decimal system is the set of whole numbers: 0, 1, 2, 3, 4, ...

Most business calculations involving whole numbers include one or more of four basic mathematical operations: addition, subtraction, multiplication, and division.

1 Read whole numbers.

What business situations require that we read and write whole numbers? Communication is one of the most important skills of successful businesspersons. Both the giver and the receiver of communications must have the same interpretation for the communication to be effective. That is why understanding terminology and the meanings of symbolic representations is an important skill.

Beginning with the ones place on the right, the place values are grouped in groups of three places. Each group of three place values is called a period. Each period has a name and a ones place, a tens place, and a hundreds place. In a number, the first period from the left may have less than three digits. In many cultures the periods are separated with commas.

Reading numbers is based on an understanding of the place-value system that is part of our decimal-number system. The chart in Figure 1-1 shows that system applied to the number 381,345,287,369,021.

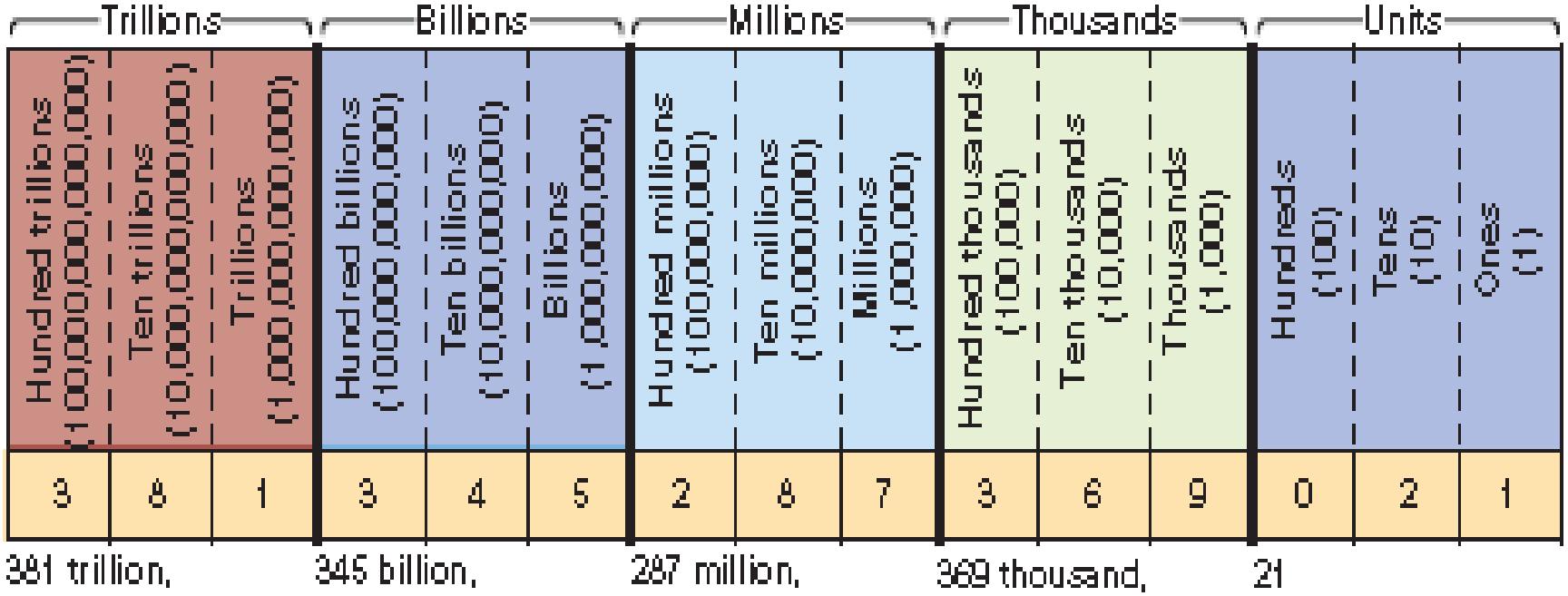


FIGURE 1-1

EXAMPLE 1

The annual operating budget for a major corporation is $3,007,047,203. Show how you would read this number

3 007 047 203

Identify each period name.

3 billion, 007 million, 047 thousand, 203

Read the words for the numbers in each period. Name each period except the units period.

Three billion, seven million, forty-seven thousand, two hun[illegible]

STOP AND CHECK

Write the words used to read the number.

1. 7,352,496

2. 4,023,508

3. 62,806,000,927

4. 587,000,000,912

2 Write whole numbers.

Suppose you are in a sales meeting and the marketing manager presents a report of the sales for the previous quarter, the projected sales for the current quarter, and the projected sales for the entire year. How would you record these figures in the notes you are taking for the meeting? You will need to have a mental picture of the place-value structure of our numbering system.

HOW TO

Write a whole number

1. Begin recording digits from left to right.
2. Insert a comma at each period name.
3. Every period after the first period must have three digits. Insert zeros as necessary.

EXAMPLE 2

In a sales presentation, Marty reported that the gross sales for the month were five hundred forty-two million, six hundred sixty-two thousand, five hundred thirty-eight. The gross sales for the previous year were fifteen billion, five hundred thousand, twenty-nine. Write these numbers in digits.

(a) Five hundred forty-two million, six hundred sixty-two thousand, five hundred thirty-eight

(b) Fifteen billion, five hundred thousand, twenty-nine

542,662,538

15,000,500,029

STOP AND CHECK

1. Write the number for eighteen billion, seventy-eight million, three hundred ninety-seven thousand, two hundred three.

3 Round whole numbers.

Exact numbers are not always necessary or desirable. For example, the board of directors does not want to know to the penny how much was spent on office supplies (although the accounting staff should know). Approximate or rounded numbers are often used. A rounded number does not represent an exact amount. It is instead an approximate number. You round a number to a specified place, which may be the first digit from the left in a number.

HOW TO

Round a whole number to a specified place

1. Find the digit in the specified place.
2. Look at the next digit to the right.
   (a) If this digit is less than 5, replace it and all digits to its right with zeros.
   (b) If this digit is 5 or more, add 1 to the digit in the specified place, and replace all digits to the right of the specified place with zeros.

Round 2,748 to the nearest hundred.

2,748

2,748

2,700

EXAMPLE 3

After the sales presentation, Marty's supervisor suggested that in future presentations, Marty use approximate numbers to illustrate the company's progress. Look at the two sales amounts in Example 2 on page 6. What are appropriate place values for rounding these numbers? Round each number to an appropriate place value.

Appropriate Rounding Places:

Large numbers are often rounded to a period place like nearest million, nearest billion, and so on.
Round the monthly sales amount to the nearest million.
Round the annual sales amount to the nearest billion.

(a) Round 542,662,538 to the nearest million.
542,662,538
543,000,000

2 is in the millions place.
The digit to the right is 6.
6 is 5 or more, so step 2b applies. Add 1 to 2 to get 3 and replace all digits to the right with zeros.

(b) Round 15,000,500,029 to the nearest billion.
15,000,500,029
15,000,000,000

5 is in the billions place.
The digit to the right is 0.
0 is less than 5, so step 2a applies. Leave [illegible] all digits to the right with zeros.

EXAMPLE 4

In making estimations it is common to round a number to the first digit from the left. Round 27,389,092 to the first digit.

27,389,092	The first digit on the left is 2.
27,389,092	The next digit to the right is 7.
30,000,000	7 is 5 or more, so step 2b applies. Increase 2 by 1 to get 3 and replace all digits to the right of 3 with zeros.

STOP AND CHECK

1. Round 3,784,921 to the nearest thousand.
2. Round 6,098 to the nearest ten.

4 Read and round integers.

In the business world and in real-life situations we sometimes want to express numbers that are smaller than 0. These numbers are negative numbers. If the temperature is lower than 0, the temperature is a negative amount. If you write a check for more than the amount of money in your bank account, your balance will be a negative number. Some business terms that often imply negative amounts are *loss* and *debt*.

The set of whole numbers is expanded by including negative whole numbers. This new set of numbers that includes whole numbers and negative whole numbers is called the set of integers. Fig 1-2 shows how the set of whole numbers is extended to include all integers. Numbers get larger as you move to the right and smaller as you move to the left. The arrows at the ends of the number line indicate that the numbers continue indefinitely in *both* directions.

FIGURE 1-2
Integers

HOW TO Read and round integers

1. For reading integers, the rules are the same as for reading whole numbers. State the word *negative* or *minus* as you begin to read a number that is less than zero. Other words such as *loss* or *debt* may be used to indicate a negative amount.
2. For rounding integers, the rules are the same as for rounding whole numbers.

EXAMPLE 5

The U.S. national debt is estimated on many different web sites. On a recent electronic counter, the national debt was given as −$11,936,042,802,503. Show how you would read this number.

−$11,936,042,802,503	Identify each period name.
Negative 11 trillion, 936 billion, 42 million, 802 thousand, 503	Read the words for the numbers in each period. Name each period except the units period.

Negative eleven trillion, nine hundred thirty-six billion, forty-two million, eight hundred two thousand, five hundred three dollars.

EXAMPLE 6

Round the U.S. national debt given in Example 5 to the nearest trillion.

−$11,936,042,802,503	The trillions digit is 1.
−$11,936,042,802,503	The digit to the right of the trillions digit is 9.
−$12,000,000,000,000	9 is more than 5, so increase 1 by 1 to get 2 and replace all digits to the right of 2 with zeros.
−$12 trillion	Sometimes in business the period name is used instead of showing all the zeros.

−$11,936,042,802,503 rounded to the nearest trillion is −$12,000,000,000,000 or −$12 trillion.

STOP AND CHECK

1. The public debt for the state of California was recently given as −$94,002,052,157. Show how you would read this number.
2. Recently the U.S. paid −$19,812,486,187 in interest on its public debt. Show how you would read this number to indicate it is being *paid out* of the national treasury.

2-1 FRACTIONS

Fractions are used to represent parts of whole items. Often fractions are implied in the narrative portion of reports and news articles. For example, a news article may claim that three out of four voters are in favor of a proposed change in a city ordinance.

1 Identify types of fractions.

We use fractions as a way to represent parts of whole numbers. If one whole quantity has four equal parts, then one of the four parts is represented by the fraction $\frac{1}{4}$ (Figure 2-1).

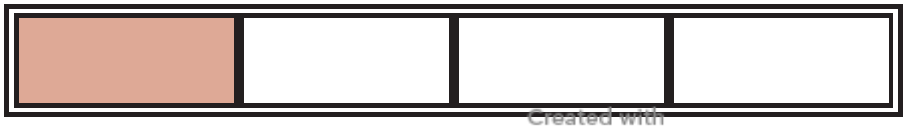


FIGURE 2-1
One part out of four parts is $\frac{1}{4}$ of the whole.

EXAMPLE 1

Visualize the fraction to identify whether it is a proper or improper fraction. Describe the relationship between the numerator and denominator.

(a) $\frac{2}{5}$ (b) $\frac{3}{2}$ (c) $\frac{4}{4}$

(a) Figure 2-2 represents $\frac{2}{5}$ or two parts out of five equal parts.

FIGURE 2-2

The fraction $\frac{2}{5}$ is a proper fraction, because it is less than one whole quantity. The numerator is smaller than the denominator.

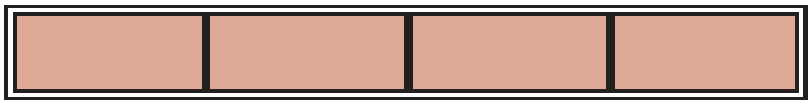
(b) Figure 2-3 represents $\frac{3}{2}$ or three parts when the one whole quantity contains two equal parts.

FIGURE 2-3

The fraction $\frac{3}{2}$ is more than one whole quantity. It is an improper fraction, because the numerator is greater than the denominator.

(c) Figure 2-4 represents $\frac{4}{4}$ or four parts when the one whole quantity contains four equal parts.

FIGURE 2-4



The fraction $\frac{4}{4}$ represents one whole quantity. It is an improper fraction, because the numerator and the denominator are equal.

2 Convert an improper fraction to a whole or mixed number.

In Figure 2-3, the fraction $\frac{3}{2}$ was shown as one whole quantity and $\frac{1}{2}$ of a second whole quantity. This amount, $\frac{3}{2}$, can also be written as $1\frac{1}{2}$. An amount written as a combination of a whole number and a fraction is called a mixed number. Every mixed number can also be written as an improper fraction.

To interpret the meaning of an improper fraction, we use its whole number or mixed number form. Thus, it is important to be able to convert between improper fractions and mixed numbers.

HOW TO

Write an improper fraction as a whole or mixed number

1. Divide the numerator of the improper fraction by the denominator.
2. Examine the remainder
   (a) If the remainder is 0, the quotient is a whole number. The improper fraction is equivalent to this whole number.
   (b) If the remainder is not 0, the quotient is not a whole number. The improper fraction is equivalent to a mixed number. The whole-number part of this mixed number is the whole-number part of the quotient. The fraction part of the mixed number has a numerator and a denominator. The numerator is the remainder; the denominator is the divisor (the denominator of the improper fraction).

Write $\frac{12}{3}$ and $\frac{13}{3}$ as whole or mixed numbers.

$$3\overline{)12}\quad 4 \qquad 3\overline{)13}\quad 4\text{ R}1$$

$$\frac{12}{3} = 4$$

$$\frac{13}{3} = 4\frac{1}{3}$$

EXAMPLE 2

Write $\frac{139}{8}$ as a whole or mixed number.

$$8\overline{)139}\quad 17\ \text{R}3,\ \text{or}\ 17\tfrac{3}{8}$$

$$\begin{array}{r} 8 \\ \hline 59 \\ 56 \\ \hline 3 \end{array}$$

Divide 139 by 8. The quotient is 17 R3, which equals $17\frac{3}{8}$.

$$\frac{139}{8} = 17\frac{3}{8}$$

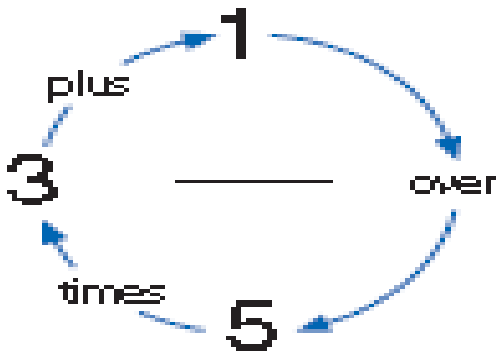
STOP AND CHECK

Write each improper fraction as a whole or mixed number.

1. $\frac{145}{28}$
2. $\frac{132}{12}$
3. $\frac{48}{12}$

3 Convert a whole or mixed number to an improper fraction.

In words, five times three plus one written over five.



In symbols,

$$3\frac{1}{5} = \frac{5 \times 3 + 1}{5} = \frac{16}{5}$$

EXAMPLE 3

Write $2\frac{3}{4}$ and 8 as improper fractions.

$$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$$

For the numerator, multiply 4 times 2 and add 3.

$$8 = \frac{8}{1}$$

Write the whole number as the numerator and 1 as the denominator

$2\frac{3}{4} = \frac{11}{4}$ and $8 = \frac{8}{1}$.

4 Reduce a fraction to lowest terms.

HOW TO

Reduce a fraction to lowest terms

1. Inspect the numerator and denominator to find any whole number that both can be evenly divided by.
2. Divide both the numerator and the denominator by that number and inspect the new fraction to find any other number that the numerator and denominator can be evenly divided by.
3. Repeat steps 1 and 2 until 1 is the only number that the numerator and denominator can be evenly divided by.

Reduce $\frac{8}{10}$ to lowest terms.

8 and 10 are divisible by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

EXAMPLE 4

Reduce $\frac{30}{36}$ to lowest terms by inspection.

$$\frac{30}{36} = \frac{30 \div 2}{36 \div 2} = \frac{15}{18}$$

Both the numerator and the denominator can be evenly divided by 2.

$$\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$$

Both the numerator and the denominator of the new fraction can be evenly divided by 3.

$\frac{30}{36}$ is reduced to $\frac{5}{6}$.

Now 1 is the only number that both the numerator and the denominator can be evenly divided by. The fraction is now in lowest terms.

HOW TO

Find the greatest common divisor of the two numbers of a proper fraction

1. Use the numerator as the first divisor and the denominator as the dividend.
2. Divide.
3. Divide the first divisor from step 2 by the remainder from step 2.
4. Divide the divisor from step 3 by the remainder from step 3.
5. Continue this division process until the remainder is 0. The last divisor is the greatest common divisor.

EXAMPLE 5

Find the GCD of 30 and 36. Then write the fraction $\frac{30}{36}$ in lowest terms.

$30\overline{)36}$ 1 R6 — Use the numerator as the first divisor and the denominator as the dividend.

$6\overline{)30}$ 5 R0 — Divide the first divisor, 30, by the first remainder, 6.

GCD = 6. — The remainder is 0, so the last divisor is the GCD.

Reduce using the GCD.

$$\frac{30}{36} = \frac{30 \div 6}{36 \div 6} = \frac{5}{6}$$ Divide the numerator and denominator by the GCD.

$\frac{30}{36}$ reduced to lowest terms is $\frac{5}{6}$.

STOP AND CHECK

1. Reduce $\frac{18}{24}$ to lowest terms by inspection.

2. Reduce $\frac{12}{36}$ to lowest terms by inspection.

1. Write $\frac{7}{12}$ as a fraction with a denominator of 36.

Write the whole or mixed number as an improper fraction.

15. $6\frac{1}{4}$

16. $27\frac{2}{5}$

2-2 ADDING AND SUBTRACTING FRACTIONS

EXAMPLE 1

Find the sum: $\frac{1}{4} + \frac{3}{4} + \frac{3}{4}$.

$$\frac{1}{4} + \frac{3}{4} + \frac{3}{4} = \frac{1+3+3}{4} = \frac{7}{4}$$

The sum of the numerators is the numerator of the sum. The original like (common) denominator is the denominator of the sum.

$$\frac{7}{4} = 1\frac{3}{4}$$

Convert the improper fraction to a whole or mixed number

The sum is $1\frac{3}{4}$.

EXAMPLE 7

Subtract $10\frac{1}{3} - 7\frac{3}{5}$.

$$10\frac{1}{3} = 10\frac{5}{15} = 9 + \frac{15}{15} + \frac{5}{15} = 9\frac{20}{15}$$

$$-7\frac{3}{5} = -7\frac{9}{15}$$

$$2\frac{11}{15}$$

Change fractions to equivalent fractions with the same LCD. Regroup in the minuend.

Subtract fractions. Subtract whole numbers.

The fraction is already in lowest terms, so you do not have to reduce it.

The difference is $2\frac{11}{15}$.

STOP AND CHECK

Subtract.

1. $\frac{7}{8} - \frac{3}{8}$

2. $\frac{5}{8} - \frac{1}{12}$

2-3 MULTIPLYING AND DIVIDING FRACTIONS

EXAMPLE 1

What fraction of the original cheese order will each of Alexa's three restaurants receive equally if $\frac{9}{10}$ of the original order is shipped?

Solution

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30}$$

Multiply numerators; multiply denominators.

$$\frac{9}{30} = \frac{3}{10}$$

Reduce to lowest terms.

EXAMPLE 2

Multiply $2\frac{1}{3} \times 3\frac{3}{4}$.

$$2\frac{1}{3} \times 3\frac{3}{4} = \frac{(3 \times 2) + 1}{3} \times \frac{(4 \times 3) + 3}{4}$$

Write the mixed numbers as improper fractions.

$$= \frac{7}{\cancel{3}_{1}} \times \frac{\overset{5}{\cancel{15}}}{4}$$

Divide both 3 and 15 by 3, reducing to 1 and 5. Multiply the numerators and denominators.

$$= \frac{35}{4} = 8\frac{3}{4}$$

Write as a mixed number.

The product is $8\frac{3}{4}$.

2 Divide fractions and mixed numbers.

EXAMPLE 5 Find the quotient: $5\frac{1}{2} \div 7\frac{1}{3}$.

$5\frac{1}{2} \div 7\frac{1}{3} =$ Write the numbers as improper fractions.

$\frac{11}{2} \div \frac{22}{3} =$ Multiply $\frac{11}{2}$ by the reciprocal of the divisor, $\frac{3}{22}$.

$\frac{\overset{1}{\cancel{11}}}{2} \times \frac{3}{\underset{2}{\cancel{22}}} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$ Reduce and multiply.

The quotient is $\frac{3}{4}$.

Divide. Write the quotient as a proper fraction or mixed number in lowest terms.

4. $\frac{7}{8} \div \frac{3}{4}$

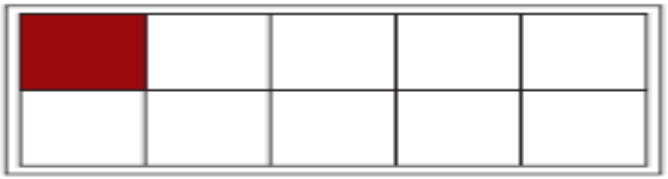
5. $2\frac{2}{5} \div 2\frac{1}{10}$

6. $3\frac{3}{8} \div 9$

3-1 DECIMALS AND THE PLACE-VALUE SYSTEM

Decimals are another way to write fractions. We use decimals in some form or another every day—even our money system is based on decimals. Calculators use decimals, and decimals are the basis of percentages, interest, markups, and markdowns.

FIGURE 3-1
1 whole divided into 10 parts. The shaded part is 0.1.



How much is 0.1? How much is 1 divided by 10? It is one part of a 10-part whole (Figure 3-1). We read 0.1 as one-tenth. Using decimal notation, we can extend our place-value chart to the right of the ones place and express quantities that are not whole numbers. When extending to the right of the ones place, a period called a **decimal point** separates the **whole-number part** from the **decimal part**.

The names of the places to the right of the decimal are tenths, hundredths, thousandths, and so on. These place names are similar to the place names for whole numbers, but they all end in *ths*. In Figure 3-2, we show the place names for the digits in the number 2,315.627432.

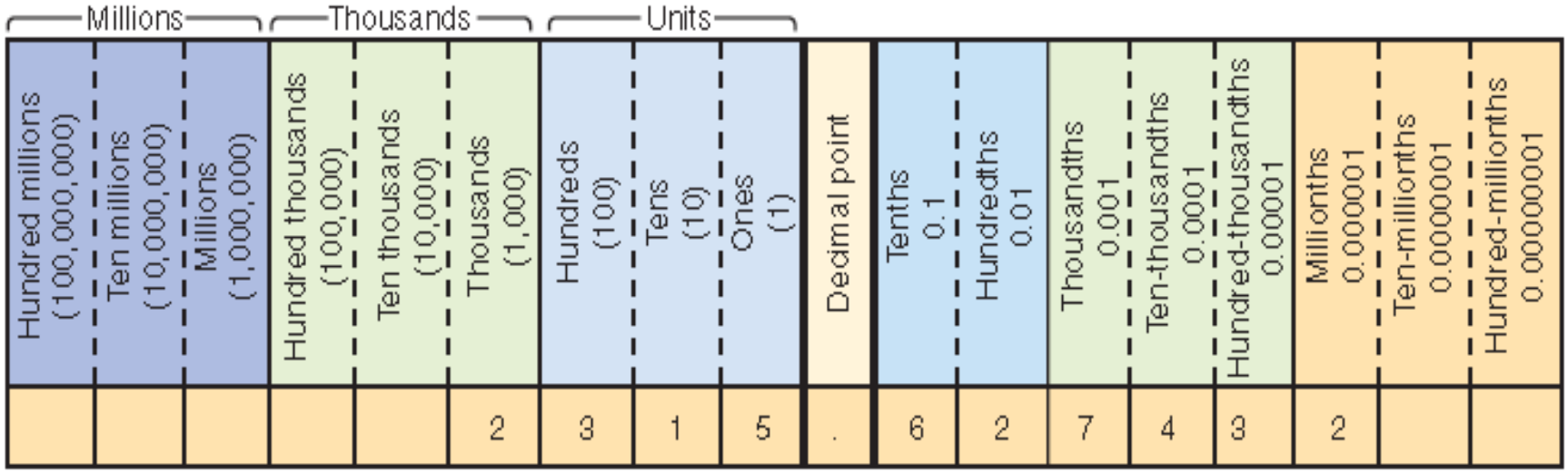


FIGURE 3-2
Place-Value Chart for Decimals

HOW TO Read or write a decimal

1. Read or write the whole-number part (to the left of the decimal point) as you would read or write a whole number.
2. Use the word *and* for the decimal point.
3. Read or write the decimal part (to the right of the decimal point) as you would read or write a whole number.
4. Read or write the place name of the rightmost digit.

Read 3.12.
Three
and
twelve
hundredths

EXAMPLE 1

Write the word name for these decimals: (a) 3.6, (b) 0.209, (c) $234.93.

(a) three and six-tenths — 3 is the whole-number part; 6 is the decimal part.
(b) two hundred nine thousandths — The whole-number part, 0, is not written.
(c) two hundred thirty-four dollars and ninety-three cents — The whole-number part is dollars. The decimal part is cents.

STOP AND CHECK

1. Write 5.8 in words.
2. Write 0.721 in words.

2 Round decimals.

As with whole numbers, we often need only an approximate amount. The process for rounding decimals is similar to rounding whole numbers.

HOW TO Round to a specified decimal place

1. Find the digit in the specified place.
2. Look at the next digit to the right.
   (a) If this digit is less than 5, eliminate it and all digits to its right.
   (b) If this digit is 5 or more, add 1 to the digit in the specified place, and eliminate all digits to its right.

Round to hundredths:
17.3754
17.3754
17.3754
17.38

EXAMPLE 2

Round the number to the specified place: (a) \$193.48 to the nearest dollar, (b) \$28.465 to the nearest cent.

(a) \$193.48 — Rounding to the nearest dollar means rounding to the ones place. The digit in the ones place is 3.

\$193.48 — The digit to the right of 3 is 4. Because 4 is less than 5, step 2a applies; eliminate 4 and all digits to its right.

\$193

\$193.48 rounded to the nearest dollar is \$193.

(b) \$28.465 — Rounding to the nearest cent means rounding to the nearest hundredth. The digit in the hundredths place is 6.

\$28.465 — The digit to the right of 6 is 5. Because 5 is 5 or more, step 2b applies.

\$28.47

\$28.465 rounded to the nearest cent is \$28.47.

STOP AND CHECK

1. Round 14.342 to the nearest tenth.

2. Round 48.7965 to the nearest hundredth.

Write the word name for the decimal.

1. 0.582
2. 0.21
3. 1.0009
4. 2.83
5. 782.07

Round to the nearest dollar.

10. \$493.91
11. \$785.03
12. \$19.80

Round to the nearest cent.

13. \$0.5239 **14.** \$21.09734 **15.** \$32,048.87219

Round to the nearest tenth.

16. 42.3784 **17.** 17.03752 **18.** 4.293

3-2 OPERATIONS WITH DECIMALS

LEARNING OUTCOMES

1. Add and subtract decimals.
2. Multiply decimals.
3. Divide decimals.

HOW TO Add or subtract decimals

1. Write the numbers in a vertical column, aligning digits according to their place values.
2. Attach extra zeros to the right end of each decimal number so that each number has the same quantity of digits to the right of the decimal point. It is also acceptable to assume blank places to be zero.
3. Add or subtract as though the numbers are whole numbers.
4. Place the decimal point in the sum or difference to align with the decimal point in the addends or subtrahend and minuend.

Add 32 + 2.55 + 8.85 + 0.625.

$$\begin{array}{r} 32 \\ 2.55 \\ 8.85 \\ 0.625 \\ \hline 44.025 \end{array}$$

EXAMPLE 1

Subtract 26.3 − 15.84.

$$\begin{array}{r} {}^{5}1{}^{12}1{}^{1}0 \\ 26.30 \\ -\ 15.84 \\ \hline 10.46 \end{array}$$

Write the numbers so that the digits align according to their place values.

Subtract the numbers, regrouping as you would in whole-number subtraction.

The difference of 26.3 and 15.84 is 10.46.

STOP AND CHECK

1. Add: 67 + 4.38 + 0.291

2. Add: 57.5 + 13.4 + 5.238

2 Multiply decimals.

HOW TO Multiply decimals

1. Multiply the decimal numbers as though they are whole numbers.
2. Count the digits in the decimal parts of both decimal numbers.
3. Place the decimal point in the product so that there are as many digits in its decimal part as there are digits you counted in step 2. If necessary, attach zeros on the left end of the product so that you can place the decimal point accurately.

Multiply 3.5×0.3

$$\begin{array}{rl} 3.5 & \text{one place} \\ \times\ 0.3 & \text{one place} \\ \hline 1.05 & \text{two places} \end{array}$$

EXAMPLE 2

Multiply 2.35×0.015.

2.35	two decimal places
$\times$ 0.015	three decimal places
1175	
235	
0.03525	five decimal places.

One 0 is attached on the left to accurately place the decimal point.

The product of 2.35 and 0.015 is 0.03525.

HOW TO

Multiply by place-value numbers such as 10, 100, and 1,000

1. Determine the number of zeros in the multiplier.
2. Move the decimal in the multiplicand to the right the same number of places as there are zeros in the multiplier. Insert zeros as necessary.

EXAMPLE 3

Multiply 36.56 by (a) 10, (b) 100, and (c) 1,000.

(a) $36.56(10) = 365.6$ Move the decimal one place to the right.

(b) $36.56(100) = 3{,}656$ Move the decimal two places to the right.

(c) $36.56(1{,}000) = 36{,}560$ Move the decimal three places to the right. Insert a zero to have enough places.

STOP AND CHECK

Multiply.

1. 4.35×0.27

2. 7.03×0.035

3 Divide decimals.

HOW TO Divide a decimal by a whole number

1. Place a decimal point for the quotient directly above the decimal point in the dividend.
2. Divide as though the decimal numbers are whole numbers.
3. If the division does not come out evenly, attach zeros as necessary and carry the division one place past the desired place of the quotient.
4. Round to the desired place.

Divide 95.2 by 14.

$$14\overline{)95.2}$$

$$\begin{array}{r} 6.8 \\ 14\overline{)95.2} \\ 84 \\ \hline 112 \\ 112 \\ \hline 0 \end{array}$$

EXAMPLE 5

Divide 5.95 by 17.

```
  0.35
17)5.95
  51
  85
  85
   0
```

Place a decimal point for the quotient directly above the decimal point in the dividend.

The quotient of 5.95 and 17 is 0.35.

EXAMPLE 6

Find the quotient of 37.4 ÷ 24 to the nearest hundredth.

```
  1.558   rounds to 1.56
24)37.400
  24
  134
  120
   140
   120
    200
    192
      8
```

Carry the division to the thousandths place, and then round to hundredths. Attach two zeros to the right of 4 in the dividend.

The quotient is 1.56 to the nearest hundredth.

HOW TO

Divide by place-value numbers such as 10, 100, and 1,000

1. Determine the number of zeros in the divisor.
2. Move the decimal in the dividend to the left the same number of places as there are zeros in the divisor. Insert zeros as necessary.

EXAMPLE 7

Divide 23.71 by (a) 10, (b) 100, and (c) 1,000.

(a) $23.71 \div 10 = 2.371$ Move the decimal one place to the left.

(b) $23.71 \div 100 = 0.2371$ Move the decimal two places to the left. It is preferred to write a zero in front of the decimal point.

(c) $23.71 \div 1{,}000 = 0.02371$ Move the decimal three places to the left. Insert a zero to have enough places.

If the divisor is a decimal rather than a whole number, we use an important fact: Multiplying both the divisor and the dividend by the same factor does not change the quotient.

We can see this by writing a division as a fraction.

$$10 \div 5 = \frac{10}{5} = 2$$

$$\frac{10}{5} \times \frac{10}{10} = \frac{100}{50} = 2$$

$$\frac{100}{50} \times \frac{10}{10} = \frac{1{,}000}{500} = 2$$

We've multiplied both the divisor and the dividend by a factor of 10, then by a factor of 10 again. The quotient is always 2.

HOW TO — Divide by a decimal

1. Change the divisor to a whole number by moving the decimal point to the right, counting the places as you go. Use a caret (^) to show the new position of the decimal point.
2. Move the decimal point in the dividend to the right as many places as you moved the decimal point in the divisor.
3. Place the decimal point for the quotient directly above the *new* decimal point in the dividend.
4. Divide as you would divide by a whole number. Carry the division one place past the desired place of the quotient. Round to the desired place.

Divide 3.4776 by 0.72.

$0.72\overset{\wedge}{)}3.47\,76$

$0.72\overset{\wedge}{)}\overline{3.47\overset{\wedge}{7}6}$

$0.72\overset{\wedge}{)}\overline{3.47\overset{\wedge}{7}6}$

$$\begin{array}{r} 4.83 \\ 0.72\overset{\wedge}{)}3.47\overset{\wedge}{7}6 \\ \underline{2\,88} \\ 59\;\;7 \\ \underline{57\;\;6} \\ 2\;\;16 \\ \underline{2\;\;16} \\ 0 \end{array}$$

EXAMPLE 8

Find the quotient of $59.9 \div 0.39$ to the nearest hundredth.

$0.39\overline{)59.90}$ $\quad 39\overset{\wedge}{)}5,990\overset{\wedge}{0}$

Move the decimal point two places to the right in both the divisor and the dividend.

$39\overline{)5,990\overset{\wedge}{0}}$

Place the decimal point for the quotient directly above the new decimal point in the dividend.

$$\begin{array}{r} 153.589 \approx 153.59 \text{ (rounded)} \\ 39\overline{)5,990.000} \\ \underline{3\,9} \\ 2\,09 \\ \underline{1\,95} \\ 140 \\ \underline{117} \\ 230 \\ \underline{195} \\ 350 \\ \underline{312} \\ 380 \\ \underline{351} \\ 29 \end{array}$$

Divide, carrying out the division to the thousandths place. Add three zeros to the right of the decimal point.

The quotient is 153.59 to the nearest hundredth.

STOP AND CHECK

Divide.

1. 100.80 ÷ 15

2. 358.26 ÷ 21

3-3 DECIMAL AND FRACTION CONVERSIONS

1 Convert a decimal to a fraction.

Decimals represent parts of a whole, just as fractions can. We can write a decimal as a fraction or a fraction as a decimal.

HOW TO Convert a decimal to a fraction

Write 0.8 as a fraction.

1. Find the denominator: Write 1 followed by as many zeros as there are places to the right of the decimal point.
   Denominator = 10
2. Find the numerator: Use the digits without the decimal point.
   $\frac{8}{10}$
3. Reduce to lowest terms and write as a whole or mixed number if appropriate.
   $\frac{4}{5}$

EXAMPLE 1

Change 0.38 to a fraction.

$$0.38 = \frac{38}{100}$$

The digits without the decimal point form the numerator.

There are two places to the right of the decimal point, so the denominator is 1 followed by two zeros.

$$\frac{38}{100} = \frac{19}{50}$$

Reduce the fraction to lowest terms.

0.38 written as a fraction is $\frac{19}{50}$.

EXAMPLE 2

Change 2.43 to a mixed number.

$$2.43 = 2\frac{43}{100}$$

The whole-number part of the decimal stays as the whole number part of the mixed number.

2.43 is $2\frac{43}{100}$ as a mixed number.

STOP AND CHECK

Write as a fraction or mixed number, and write in simplest form.

1. 0.7 **2.** 0.32 **3.** 2.087

2 Convert a fraction to a decimal.

Fractions indicate division. Therefore, to write a fraction as a decimal, perform the division. Divide the numerator by the denominator, as you would divide decimals.

HOW TO Write a fraction as a decimal

1. Write the numerator as the dividend and the denominator as the divisor.
2. Divide the numerator by the denominator. Carry the division as many decimal places as necessary or desirable.
3. For repeating decimals:
   (a) Write the remainder as the numerator of a fraction and the divisor as the denominator.
   or
   (b) Carry the division one place past the desired place and round.

EXAMPLE 3

Change $\frac{1}{4}$ to a decimal number.

$$
\begin{array}{r}
0.25 \\
4\overline{)1.00} \\
\underline{8} \\
20 \\
\underline{20}
\end{array}
$$

Divide the numerator by the denominator, adding zeros to the right of the decimal point as needed.

The decimal equivalent of $\frac{1}{4}$ is 0.25.

When the division comes out even (there is no remainder), we say the division terminates, and the quotient is called a **terminating decimal.** If, however, the division *never* comes out even (there is always a remainder), we call the number a **nonterminating** or **repeating decimal.** If the quotient is a repeating decimal, either write the remainder as a fraction or round to a specified place.

EXAMPLE 4

Write $\frac{2}{3}$ as a decimal number in hundredths (a) with the remainder expressed as a fraction and (b) with the decimal rounded to hundredths.

(a)
$$\begin{array}{r} 0.66\frac{2}{3} \\ 3\overline{)2.00} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

(b)
$$\begin{array}{r} 0.666 \approx 0.67 \\ 3\overline{)2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$\frac{2}{3} = 0.66\frac{2}{3}$ **or** $\frac{2}{3} \approx 0.67$.

EXAMPLE 5

Write $3\frac{1}{4}$ as a decimal.

$3\frac{1}{4} = 3.25$

The whole-number part of the mixed number stays as the whole-number part of the decimal number.

$3\frac{1}{4}$ is 3.25 as a decimal number.

STOP AND CHECK

Change to decimal numbers. Round to hundredths if necessary.

1. $\frac{3}{5}$
2. $\frac{7}{8}$
3. $\frac{5}{12}$

USING EQUATIONS TO SOLVE PROBLEMS

1 Use the problem-solving approach to analyze and solve word problems.

Certain key words in a problem give you clues as to whether a certain quantity is added to subtracted from, or multiplied or divided by another quantity. For example, if a word problem tells you that Carol's salary in 2011 *exceeds* her 2010 salary by $2,500, you know that you should *add* $2,500 to her 2010 salary to find her 2011 salary. Many times, when you see the word *of* in a problem, the problem involves multiplication. Table 5-1 summarizes important key words and what they generally imply when they are used in a word problem. This list should help you analyze the information in word problems and write the information in symbols.

We can relate the steps in our five-step problem-solving approach to writing and solving equations.

What You Know	Known or given facts
What You Are Looking For	Unknown amounts (Assign a letter to represent an unknown amount. Other unknown amounts are written related to the assigned letter.)
Solution Plan	Equation or relationship among the known and unknown facts
Solution	Solving the equation
Conclusion	Solution interpreted within the context of the problem

EXAMPLE 1

Full-time employees at Charlie's Steakhouse work more hours per day than part-time employees. If the difference of working hours is 4 hours per day, and if part-timers work 6 hours per day, how many hours per day do full-timers work?

What You Know	What You Are Looking For	Solution Plan
Hours per day that part-timers work: 6 Difference between hours worked by full-timers and hours worked by part-timers: 4	Hours per day that full-timers work: N	The word *difference* implies subtraction. Full-time hours − part-time hours = difference of hours $N - 6 = 4$

Solution

$$\begin{array}{rcl} N - 6 & = & 4 \\ +\,6 & & +\,6 \\ \hline N & = & 10 \end{array}$$

Undo subtraction.

The solution is 10.

Check:

$10 - 6 \stackrel{?}{=} 4$ Replace N with 10. Subtract.

$4 = 4$ The sides are equal.

Conclusion

The hours per day that full-timers work is 10.

EXAMPLE 2

Wanda plans to save $\frac{1}{10}$ of her salary each week. If her weekly salary is \$350, how much will she save each week?

What You Know	What You Are Looking For	Solution Plan
Salary = \$350 Rate of saving: $\frac{1}{10}$	Amount to be saved: S	The word *of* implies multiplication. Amount to be saved = rate of saving $\times$ salary $S = \frac{1}{10}(\$350)$

Solution

$S = \frac{1}{\cancel{10}_{1}}(\$\cancel{350}^{35})$ Reduce and multiply.

$S = \$35$ The solution is 35.

Check:

$\$35 \stackrel{?}{=} \frac{1}{\cancel{10}_{1}}(\$\cancel{350}^{35})$ Replace S with \$35 and see if the sides are equal.

$\$35 = \$35$

Conclusion

Wanda will save \$35 per week.

EXAMPLE 5

Your car gets 23 miles to a gallon of gas. How far can you go on 16 gallons of gas?

What You Know	What You Are Looking For	Solution Plan
Distance traveled using 1 gallon: 23 miles (Pair 1)	Distance traveled using 16 gallons: M miles (Pair 2)	Miles traveled per 16 gallons is proportional to miles traveled for each 1 gallon. $\frac{1 \text{ gallon}}{23 \text{ miles}} = \frac{16 \text{ gallons}}{M \text{ miles}}$ Pair 1 Pair 2

Solution

$$\frac{1}{23} = \frac{16}{M}$$ Cross multiply.

$$1M = (16)(23)$$ Multiply.

$$M = 368$$

Check:

$$\frac{1}{23} \stackrel{?}{=} \frac{16}{368}$$ Substitute 368 for M and cross multiply.

$$(1)(368) \stackrel{?}{=} (23)(16)$$ Multiply.

$$368 = 368$$

Conclusion

You can travel 368 miles using 16 gallons of gas.

STOP AND CHECK

1. Carrie McConnell spends $\frac{1}{6}$ of her weekly earnings on groceries. What are her weekly earnings if she spends $117.50 on groceries each week?

2. Marcus James purchased 2,500 pounds of produce. Records indicate he purchased 800 pounds of potatoes, 150 pounds of broccoli, and 390 pounds of tomatoes. He also purchased apples. How many pounds of apples did he purchase?

3. Hilton Hotel has 8 times as many nonsmoking rooms as it has smoking rooms. If the hotel has 873 rooms in its inventory, how many are smoking rooms?

EXAMPLE 1

Wal-Mart purchases a Sony plasma television for $875 and marks it up $400. What is the selling price of the television? Use the formula $S = C + M$, where S is the selling price, C is the cost, and M is the markup.

$S = C + M$ Write the formula. Substitute known values for C and M.

$S = \$875 + \$400$ Add.

$S = \$1{,}275$

The selling price for the television is $1,275.

In some instances, the missing value is not the value that is isolated in the formula. After the known values are substituted into the formula, use the techniques for solving equations to find the missing value.

EXAMPLE 2

A DVD player that costs $85 sells for $129. What is the markup on the player? Use the formula $S = C + M$, where S is the selling price, C is the cost, and M is the markup.

$S = C + M$ Write the formula. Substitute known values for C and S.

$\$129 = \$85 + M$ Subtract $85 from each side of the equation.

$-85 \quad -85$

$\$44 = M$

The markup for the DVD player is $44.

PERCENT EQUIVALENTS

1 Write a whole number, fraction, or decimal as a percent.

The businessperson must be able to write whole numbers, decimals, or fractions as percents, and to write percents as whole numbers, decimals, or fractions. First we examine writing whole numbers, decimals, and fractions as percents.

Hundredths and percent have the same meaning: per hundred. Just as 100 cents is the same as 1 dollar, 100 percent is the same as 1 whole quantity.

$$100\% = 1$$

This fact is used to write percent equivalents of numbers, and to write numerical equivalents of percents. It is also used to calculate markups, markdowns, discounts, and numerous other business applications.

$$1 = \frac{2}{2} \quad \text{and} \quad \frac{1}{2}\left(\frac{2}{2}\right) = \frac{2}{4}$$

We can also use the fact that $N \times 1 = N$ to change numbers to equivalent percents.

$$1 = 100\% \qquad \frac{1}{2} = \frac{1}{2}(100\%) = \frac{1}{2}\left(\frac{100\%}{1}\right) = 50\%$$

$$0.5 = 0.5(100\%) = 50\%$$

HOW TO

Write a number as its percent equivalent

1. Multiply the number by 1 in the form of 100%.
2. The product has a % symbol.

Write 0.3 as a percent.
$0.3 = 0.3(100\%) =$
$030.\% = 30\%$

EXAMPLE 1

Write the decimal or whole number as a percent.

(a) 0.27 (b) 0.875 (c) 1.73 (d) 0.004 (e) 2

(a) $0.27 = 0.27(100\%) = 027.\% = 27\%$
0.27 as a percent is 27%.
Multiply 0.27 by 100% (move the decimal point two places to the right).

(b) $0.875 = 0.875(100\%) = 087.5\% = 87.5\%$
0.875 as a percent is 87.5%.
Multiply 0.875 by 100% (move the decimal point two places to the right).

(c) $1.73 = 1.73(100\%) = 173.\% = 173\%$
1.73 as a percent is 173%.
Multiply 1.73 by 100% (move the decimal point two places to the right).

(d) $0.004 = 0.004(100\%) = 000.4\% = 0.4\%$
0.004 as a percent is 0.4%
Multiply 0.004 by 100% (move the decimal point two places to the right).

(e) $2 = 2(100\%) = 200.\% = 200\%$
2 as a percent is 200%.
Multiply 2 by 100% (move the decimal point two places to the right).

EXAMPLE 2

Write the fraction as a percent.

(a) $\frac{67}{100}$ (b) $\frac{1}{4}$ (c) $3\frac{1}{2}$ (d) $\frac{7}{4}$ (e) $\frac{2}{3}$

(a) $\frac{67}{100} = \frac{67}{\cancel{100}}\left(\frac{\cancel{100}\%}{1}\right) = \mathbf{67\%}$ Reduce and multiply.

(b) $\frac{1}{4} = \frac{1}{\cancel{4}}\left(\frac{\cancel{100}\%}{1}\right) = \mathbf{25\%}$ Reduce and multiply.

(c) $3\frac{1}{2} = 3\frac{1}{2}\left(\frac{100\%}{1}\right) = \frac{7}{\cancel{2}}\left(\frac{\cancel{100}\%}{1}\right) = \mathbf{350\%}$ Change to an improper fraction, reduce, and multiply.

(d) $\frac{7}{4} = \frac{7}{\cancel{4}}\left(\frac{\cancel{100}\%}{1}\right) = \mathbf{175\%}$ Reduce and multiply.

(e) $\frac{2}{3} = \frac{2}{3}\left(\frac{100\%}{1}\right) = \frac{200\%}{3} = \mathbf{66\frac{2}{3}\%}$ Multiply.

STOP AND CHECK

Write the decimal or whole number as a percent.

1. 0.82 **2.** 3.45 **3.** 0.0007

2 Write a percent as a whole number, fraction, or decimal.

When a number is divided by 1, the quotient has the same value as the original numbe $N \div 1 = N$ or $\frac{N}{1} = N$. We have used this concept to reduce fractions. For example,

$$1 = \frac{2}{2} \qquad \frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$$

We can also use the fact that $N \div 1 = N$ or $\frac{N}{1} = N$ to change percents to numerical equivalent

$$50\% \div 100\% = \frac{50\%}{100\%} = \frac{50}{100} = \frac{1}{2}$$

$$50\% \div 100\% = 50 \div 100 = 0.50 = 0.5$$

HOW TO Write a percent as a number

1. Divide the number by 1 in the form of 100% or multiply by $\frac{1}{100\%}$.
2. The quotient does not have a % symbol.

EXAMPLE 3

Write the percent as a decimal.

(a) 37% (b) 26.5% (c) 127% (d) 7% (e) 0.9% (f) $2\frac{19}{20}\%$ (g) $167\frac{1}{3}\%$

(a) $37\% = 37\% \div 100\% = 0.37 = \mathbf{0.37}$ Divide by 100 mentally.

(b) $26.5\% = 26.5\% \div 100\% = 0.265 = \mathbf{0.265}$ Divide by 100 mentally.

(c) $127\% = 127\% \div 100\% = 1.27 = \mathbf{1.27}$ Divide by 100 mentally.

(d) $7\% = 7\% \div 100\% = 0.07 = \mathbf{0.07}$ Divide by 100 mentally.

(e) $0.9\% = 0.9\% \div 100\% = 0.009 = \mathbf{0.009}$ Divide by 100 mentally.

(f) $2\frac{19}{20}\% = 2.95\% \div 100\% = 0.0295 = \mathbf{0.0295}$ Write the mixed number in front of the percent symbol as a mixed decimal before dividing by 100%.

(g) $167\frac{1}{3}\% = 167.3\overline{3}\% \div 100\%$
$= 1.673\overline{3} = \mathbf{1.673\overline{3}}$ **or 1.673 (rounded)** Write the mixed number in front of the percent symbol as a repeating decimal before dividing by 100.

EXAMPLE 4

Write the percent as a fraction or mixed number.

(a) 65% (b) $\frac{1}{4}\%$ (c) 250% (d) $83\frac{1}{3}\%$ (e) 12.5%

(a) $65\% = 65\% \div 100\% = \frac{65\%}{1}\left(\frac{1}{100\%}\right) = \mathbf{\frac{13}{20}}$ Convert division to multiplication.

(b) $\frac{1}{4}\% = \frac{1}{4}\% \div 100\% = \frac{1\%}{4}\left(\frac{1}{100\%}\right) = \mathbf{\frac{1}{400}}$

(c) $250\% = 250\% \div 100\% = \frac{250\%}{1}\left(\frac{1}{100\%}\right) = \frac{5}{2} = \mathbf{2\frac{1}{2}}$

STOP AND CHECK

Write the percent as a decimal.

1. 52% **2.** 38.5% **3.** 143% **4.** 0.72%

6-1 PERCENT EQUIVALENTS

With fractions and decimals, we compare only like quantities, that is, fractions with common denominators and decimals with the same number of decimal places. We can standardize our representation of quantities so that they can be more easily compared. We standardize by expressing quantities in relation to a standard unit of 100. This relationship, called a **percent**, is used to solve many different types of business problems.

The word *percent* means *hundredths* or *out of 100* or *per 100* or *over 100* (in a fraction). That is, 44 percent means 44 hundredths, or 44 out of 100, or 44 per 100, or 44 over 100. We can write 44 hundredths as 0.44 or $\frac{44}{100}$.

The symbol for *percent* is %. You can write 44 percent using the percent symbol: 44%; using fractional notation: $\frac{44}{100}$; or using decimal notation: 0.44.

$$44\% = 44 \text{ percent} = 44 \text{ hundredths} = \frac{44}{100} = 0.44$$

1 Write a whole number, fraction, or decimal as a percent.

The businessperson must be able to write whole numbers, decimals, or fractions as percents, and to write percents as whole numbers, decimals, or fractions. First we examine writing whole numbers, decimals, and fractions as percents.

Hundredths and percent have the same meaning: per hundred. Just as 100 cents is the same as 1 dollar, 100 percent is the same as 1 whole quantity.

$$100\% = 1$$

This fact is used to write percent equivalents of numbers, and to write numerical equivalents of percents. It is also used to calculate markups, markdowns, discounts, and numerous other business applications.

When we multiply a number by 1, the product has the same value as the original number. $N \times 1 = N$. We have used this concept to change a fraction to an equivalent fraction with a higher denominator. For example,

$$1 = \frac{2}{2} \quad \text{and} \quad \frac{1}{2}\left(\frac{2}{2}\right) = \frac{2}{4}$$

We can also use the fact that $N \times 1 = N$ to change numbers to equivalent percents.

$$1 = 100\% \qquad \frac{1}{2} = \frac{1}{2}(100\%) = \frac{1}{2}\left(\frac{100\%}{1}\right) = 50\%$$

$$0.5 = 0.5(100\%) = 050.\% = 50\%$$

EXAMPLE 1

Write the decimal or whole number as a percent.

(a) 0.27 (b) 0.875 (c) 1.73 (d) 0.004 (e) 2

(a) $0.27 = 0.27(100\%) = 027.\% = 27\%$

0.27 as a percent is 27%.

Multiply 0.27 by 100% (move the decimal point two places to the right).

(b) $0.875 = 0.875(100\%) = 087.5\% = 87.5\%$

0.875 as a percent is 87.5%.

Multiply 0.875 by 100% (move the decimal point two places to the right).

(c) $1.73 = 1.73(100\%) = 173.\% = 173\%$

1.73 as a percent is 173%.

Multiply 1.73 by 100% (move the decimal point two places to the right).

EXAMPLE 2

Write the fraction as a percent.

(a) $\frac{67}{100}$ (b) $\frac{1}{4}$ (c) $3\frac{1}{2}$ (d) $\frac{7}{4}$ (e) $\frac{2}{3}$

(a) $\frac{67}{100} = \frac{67}{100}\left(\frac{100\%}{1}\right) = 67\%$

Reduce and multiply.

(c) $3\frac{1}{2} = 3\frac{1}{2}\left(\frac{100\%}{1}\right) = \frac{7}{2}\left(\frac{100\%}{1}\right) = \mathbf{350\%}$ Change to an improper fraction, reduce, and multiply.

(d) $\frac{7}{4} = \frac{7}{4}\left(\frac{100\%}{1}\right) = \mathbf{175\%}$ Reduce and multiply.

(e) $\frac{2}{3} = \frac{2}{3}\left(\frac{100\%}{1}\right) = \frac{200\%}{3} = \mathbf{66\frac{2}{3}\%}$ Multiply.

STOP AND CHECK

Write the decimal or whole number as a percent.

1. 0.82 **2.** 3.45 **3.** 0.0007 **4.** 5

2 Write a percent as a whole number, fraction, or decimal.

When a number is divided by 1, the quotient has the same value as the original number. $N \div 1 = N$ or $\frac{N}{1} = N$. We have used this concept to reduce fractions. For example,

$$1 = \frac{2}{2} \qquad \frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$$

We can also use the fact that $N \div 1 = N$ or $\frac{N}{1} = N$ to change percents to numerical equivalents.

$$50\% \div 100\% = \frac{50\%}{100\%} = \frac{50}{100} = \frac{1}{2}$$

$$50\% \div 100\% = 50 \div 100 = 0.50 = 0.5$$

HOW TO — Write a percent as a number

1. Divide the number by 1 in the form of 100% or multiply by $\frac{1}{100\%}$.
2. The quotient does not have a % symbol.

EXAMPLE 3

Write the percent as a decimal.

(a) 37% (b) 26.5% (c) 127% (d) 7% (e) 0.9% (f) $2\frac{19}{20}\%$ (g) $167\frac{1}{3}\%$

(a) $37\% = 37\% \div 100\% = 0.37 = \mathbf{0.37}$ Divide by 100 mentally.

(b) $26.5\% = 26.5\% \div 100\% = 0.265 = \mathbf{0.265}$ Divide by 100 mentally.

(c) $127\% = 127\% \div 100\% = 1.27 = \mathbf{1.27}$ Divide by 100 mentally.

EXAMPLE 4

Write the percent as a fraction or mixed number.

(a) 65% (b) $\frac{1}{4}\%$ (c) 250% (d) $83\frac{1}{3}\%$ (e) 12.5%

(a) $65\% = 65\% \div 100\% = \frac{65\%}{1}\left(\frac{1}{100\%}\right) = \mathbf{\frac{13}{20}}$ Convert division to multiplication.

(b) $\frac{1}{4}\% = \frac{1}{4}\% \div 100\% = \frac{1\%}{4}\left(\frac{1}{100\%}\right) = \mathbf{\frac{1}{400}}$

(c) $250\% = 250\% \div 100\% = \frac{250\%}{1}\left(\frac{1}{100\%}\right) = \frac{5}{2} = \mathbf{2\frac{1}{2}}$

STOP AND CHECK

Write the percent as a decimal.

1. 52% **2.** 38.5% **3.** 143% **4.** 0.72%

6-2 SOLVING PERCENTAGE PROBLEMS

1 Identify the rate, base, and portion in percent problems.

A **formula** expresses a relationship among quantities. When you use the five-step problem-solving approach, the third step, the Solution Plan, is often a formula written in words and letters.

The percentage formula, Portion = Rate × Base, can be written as $P = RB$. The letters or words represent numbers.

In the formula $P = RB$, the **base** (B) represents the original number or one entire quantity. The **portion** (P) represents a part of the base. The **rate** (R) is a percent that tells us how the base and portion are related. In the statement "50 is 20% of 250," 250 is the base (the entire quantity), 50 is the portion (part), and 20% is the rate (percent).

EXAMPLE 1

Identify the given and missing elements for each example.

(a) 20% of 75 is what number?
(b) What percent of 50 is 30?
(c) Eight is 10% of what number?

R *B* *P*
(a) 20% of 75 is what number?
Percent Total Part

R *B* *P*
(b) What percent of 50 is 30?
Percent Total Part

P *R* *B*
(c) Eight is 10% of what number?
Part Percent Total

Use the identifying key words for rate (*percent* or %), base (*total*, *original*, associated with the word *of*), and portion (*part*, associated with the word *is*).

STOP AND CHECK

Identify the base, rate, and portion.

1. 42% of 85 is what number?
2. Fifty is 15% of what number?
3. What percent of 80 is 20?

2 Use the percentage formula to find the unknown value when two values are known.

The percentage formula, Portion = Rate × Base, can be written as $P = RB$. When the numbers are put in place of the letters, the formula guides you through the calculations.

The three forms of the percentage formula are

Portion = Rate × Base	$P = RB$	For finding the portion.
$\text{Base} = \frac{\text{Portion}}{\text{Rate}}$	$B = \frac{P}{R}$	For finding the base.
$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$	$R = \frac{P}{B}$	For finding the rate.

Circles can help us visualize these formulas. The shaded part of each circle in Figure 6-1 represents the missing amount. The unshaded parts represent the known amounts. If the unshaded parts are *side by side*, *multiply* their corresponding numbers to find the unknown number. If the unshaded parts are *one on top of the other*, *divide* the corresponding numbers to find the unknown number.

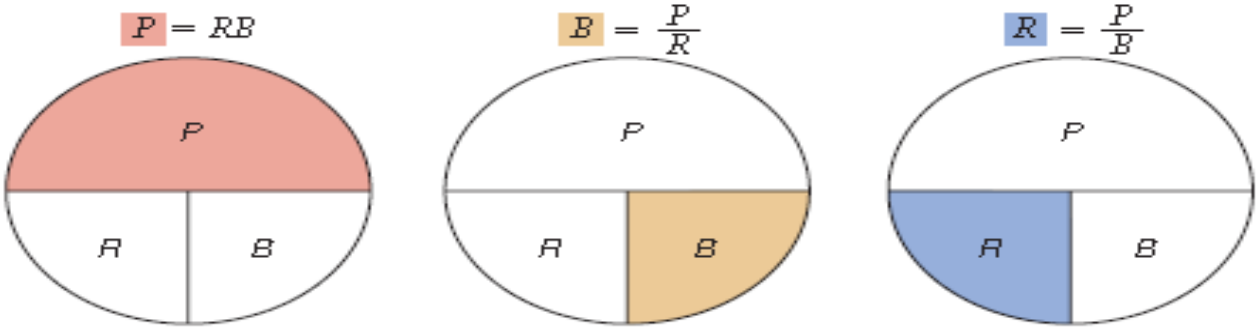


FIGURE 6-1
Forms of the Percentage Formula

HOW TO Use the percentage formula to solve percentage problems

1. Identify and classify the two known values and the one unknown value.
2. Choose the appropriate percentage formula for finding the unknown value.
3. Substitute the known values into the formula. For the rate, use the decimal or fractional equivalent of the percent.
4. Perform the calculation indicated by the formula.
5. Interpret the result. If finding the rate, convert decimal or fractional equivalents of the rate to a percent.

EXAMPLE 2

Solve the problems.

(a) 20% of 400 is what number?
(b) 20% of what number is 80?
(c) 80 is what percent of 400?

(a) 20% = Rate — Identify known values and unknown value.
400 = Base
Portion is unknown
$P = RB$ — Choose the appropriate formula.
$P = 0.2(400)$ — Substitute values using the decimal equivalent of 20%.
$P = \mathbf{80}$ — Perform calculation.
20% of 400 is 80. — Interpret result.

(b) 20% = Rate — Identify known values and unknown value.
80 = Portion
Base is unknown
$B = \frac{P}{R}$ — Choose the appropriate formula.
$B = \frac{80}{0.2}$ — Substitute values. Perform calculation.
$B = \mathbf{400}$
20% of 400 is 80. — Interpret result.

EXAMPLE 3

During a special one-day sale, 600 customers bought the on-sale pizza. Of these customers, 20% used coupons. The manager will run the sale again the next day if more than 100 coupons were used. Should she run the sale again?

What You Know	What You Are Looking For	Solution Plan
Total customers: 600 Coupon-using customers as a percent of total customers: 20%	Quantity of coupon-using customers Should the manager run the sale again?	The quantity of coupon-using customers is a *portion* of the *base* of total customers, at a *rate* of 20% (Figure 6-2). $P = RB$ Quantity of coupon-using customers $= RB$

Solution

$P = RB$	P is unknown; $R = 20\%; B = 600$
$P = 20\%(600)$	Substitute known values. Change % to decimal equivalent.
$P = 0.2(600)$	Multiply.
$P = 120$	

Conclusion

The quantity of coupon-using customers is 120.
Because 120 is more than 100, the manager should run the sale again.

EXAMPLE 4

If $66\frac{2}{3}\%$ of the 900 employees in a company choose the Preferred Provider insurance plan, how many people from that company are enrolled in the plan?

First, identify the terms. The rate is the percent, and the base is the total number of employees. The portion is the quantity of employees enrolled in the plan.

$P = RB$ — The portion is the unknown value (Figure 6-3).

$P = 66\frac{2}{3}\%(900)$ — The rate is $66\frac{2}{3}\%$; the base is 900. Write $66\frac{2}{3}\%$ as a fraction.

$P = \frac{2}{3}\left(\frac{900}{1}\right) = 600$ — Multiply.

The Preferred Provider plan has 600 people enrolled.

EXAMPLE 5

Stan sets aside 15% of his weekly income for rent. If he sets aside $150 each week, what is his weekly income?

Identify the terms: The rate is the number written as a percent, 15%. The portion is given, $150 it is a portion of his weekly income, the unknown base.

$B = \frac{P}{R}$ — The rate is 15% and the portion is $150 (Figure 6-4). The base is the weekly income to be found.

$B = \frac{\$150}{15\%}$ — Convert 15% to a decimal equivalent.

$B = \frac{150}{0.15}$ — Divide.

$B = \$1{,}000$

Stan's weekly income is $1,000.

STOP AND CHECK

1. 15% of 200 is what number?
2. 25% of what number is 120?
3. 150 is what percent of 750?

6-2 SECTION EXERCISES

SKILL BUILDERS

Identify the rate, base, and portion.

1. 48% of 12 is what number?
2. 32% of what number is 28?
3. What percent of 158 is 47.4?

6-3 INCREASES AND DECREASES

1 Find the amount of increase or decrease in percent problems.

The amount of increase or decrease is the amount that an original number changes. Subtraction is used to find the amount of change when the beginning and ending (or new) amounts are known.

HOW TO

Find the amount of increase or decrease from the beginning and ending amounts

1. To find the amount of increase (when new amount is larger than beginning amount):

   Amount of increase = new amount − beginning amount

2. To find the amount of decrease (when new amount is smaller than beginning amount):

   Amount of decrease = beginning amount − new amount

EXAMPLE 1

David Spear's salary increased from $58,240 to $63,190. What is the amount of increase?

$$\begin{aligned} \text{Beginning amount} &= \$58{,}240 \\ \text{New amount} &= \$63{,}190 \\ \text{Increase} &= \text{new amount} - \text{beginning amount} \\ &= \$63{,}190 - \$58{,}240 \\ &= \$4{,}950 \end{aligned}$$

David's salary increase was $4,950.

EXAMPLE 2

A coat was marked down from $98 to $79. What is the amount of markdown?

$$\begin{aligned} \text{Beginning amount} &= \$98 \\ \text{New amount} &= \$79 \\ \text{Decrease} &= \text{beginning amount} - \text{new amount} \\ &= \$98 - \$79 \\ &= \$19 \end{aligned}$$

The coat was marked down $19.