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Soft Sets Theory

This chapter will discuss what is nowadays called *soft set theory*. in 1999 the notion of soft set theory is given by Dmitriy Molodtsov [24] and this is so useful to solve or to find the solution of many complicated problems in engineering, environment, and economics. He has established the fundamental results of soft set theory and successfully applied the soft set theory into several directions, like theory of probability, operations research, smoothness of functions, game theory and so on. Soft set theory has a wider application and its progress is useful and very rapid in different fields. The concept of generalized closed sets in general topology is given by Norman Levine [4]. Kannan [22] introduced soft generlized closed sets and open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Also, some of their properties are studied. Next, Yuksel et al. [26] studied behavior relative to soft subspaces of soft generalized closed sets and continued investigating the characteristics of soft generalized closed and open sets. They established their several properties in a soft compact (soft regular space, soft normal space, soft Lindelof space, soft countably compact space). The concept of soft topological spaces is presented by Muhammad and Munazza [25] and then some notions of the soft set (F,A) like soft closure and soft interior points of soft set (F,A) which are denoted by $cl(F,A)$ and $int(F,A)$ respectively, also soft open sets, soft closed sets, soft neighborhood of a point, and soft separation axioms are introduced and studied in soft topological spaces. Soft semi-open sets and its characteristics were introduced and discussed by Bin [27]. In section one, some basic definitions have been used to obtain the results and properties developed in this Chapter are presented. In section two, the class of generalized soft open sets in soft topological spaces, namely soft $b-$open sets, is introduced and studied. Then discussed the relationships among soft α-open sets, soft semi-open sets, soft pre-open sets and soft $b-$open sets. Further, we investigated the concepts of soft $b-$open functions and soft $b-$continuous functions and discussed their relationships with soft continuous and other weaker forms of soft continuous functions. In section three, the concepts of soft πGR-closed and soft $\pi GR\text{-}T_{1/2}$ – spaces are introduced and discussed.

1. Preliminaries

We now begin by recalling some definitions and some of the basic prosperities of the soft sets.

Definition 1.1:

Let F be a multi-valued function $F : A \to P(U)$ where $P(U)$ is a power set of a universe U and A is a subset of a parameter set E. Then a pair (F, A) is called a soft set over U. Further, we consider that a parameterized collection of subsets of the set U is the soft set. Any set $F(e)$, $e \in E$, from this class may be determined as the set of $e-$approximate elements of the soft set (F, A) or as the set of e-elements of the soft set. In other words, a soft set is not a set. Further, let (F, A) and (G, B) be two soft sets over the common universe U. Hence (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations and hence we write $(F, A) \tilde{\subseteq} (G, B)$. Also, if $(F, A) \tilde{\subseteq} (G, B)$, then (F, A) is said to be a soft superset of (G, B). Moreover, (F, A) and (G, B) are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A). If $F(e) = \phi, \forall e \in A$, then a soft set (F, A) over U is called a null soft set, denoted by $\Phi = (\phi, \phi)$. Also, if $F(e) = U, \forall e \in A$, then a soft set (F, A) over U is said to be universal soft set and denoted by (U, E). The family of all soft sets (F, A) over a universe U and the parameter set E is denoted by $SS(U_E)$.

Definition 1.2: Let (F, A) and (G, B) be two soft sets over X, then their union is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $G(e)$

if $e \in B - A$, $F(e) \bigcup G(e)$ if $e \in A \bigcap B$. We write $(F, A) \coprod (G, B) = (H, C)$. Also, $(F, A) \prod (G, B) = (H, C)$, where $C = A \bigcap B$ and $H(e) = F(e) \bigcap G(e)$ for all $e \in C$.

Definition 1.3: Let $(F, A) \in SS(U_A)$. Then (F,A) is called a soft point in (U, A), and denoted by e_F , if $F(e') = \phi$ for all $e' \in A - \{e\}$ while for the element $e \in A$, $F(e) \neq \phi$. Further, if $F(e) \subseteq G(e)$, for the element $e \in A$. Then the soft point e_F belongs to the soft set (G, A) and denoted by $e_F \tilde{\in} (G, A)$,

Definition 1.4: The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \setminus (G, E)$, is defined as $H(e) = F(e) \setminus G(e)$, for all $e \in E$.

Definition 1.5: Let (F, A) be a soft set over X. The complement of (F, A) with respect to the universal soft set (X, E), denoted by $(F, A)^c$, is defined as (F^c, D), where $D = E \setminus \{e \in A \mid F(e) = X\} = \{e \in A \mid F(e) = X\}^c$, and for all $e \in D$,

$$F^c(e) = \begin{cases} X \setminus F(e) \, , \; if \;\; e \in A \\ X \, , \;\; Otherwise \;\; . \end{cases}$$

Proposition 1.6: Let (F, E) and (G, E) be the soft sets over X. Then

(1) $((F, E) \coprod (G, E))^C = (F, E)^C \prod (G, E)^C$

(2) $((F, E) \prod (G, E))^C = (F, E)^C \coprod (G, E)^C$.

Definition 1.7:

A soft topological space (STS in short) on a non empty set X is a family τ of soft sets over the common universe X satisfying the following axioms:.

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(i) Φ and (X, E) are belong to τ .

(ii) The union of any random number of soft sets in τ belongs to τ .

(iii) The intersection of any two soft sets in τ belongs to τ .

The triplet (X, E, τ) is said to be a soft topological space over X . Each member in τ is called soft open set in X and its complement is called soft closed set in X .

Definition 1.8:

The soft closure of (F, A) is denoted by (F, A). Where (F, A) is the intersection of all soft closed sets containing (F, A). Therefore (F, A) is the smallest soft closed set such that $(F, A) \tilde{\subseteq}$ (F, A), and the soft interior of (F, A) is denoted by (F, A). Where (F, A) is the union of all soft open set is contained in (F, A). Therefore (F, A) is the biggest soft open set such that $\text{int}^S (F, A) \tilde{\subseteq} (F, A)$. Also, we define soft regular closure, soft α-closure, soft pre-closure, soft semi closure, and soft semi pre-closure of the soft set (F, A) of a soft topological space X and are denoted by (F, A), α (F, A), p (F, A), s (F, A), and (F, A) respectively. The family of all soft α-open (resp. soft preopen, soft semi-open, soft semi-preopen, soft regular open) sets in a soft topological space (X, E, τ) is denoted by $\tau^{s\alpha}$ (resp. SPO(X , τ), SSO(X , τ), SSPO(X , τ), SRO(X , τ)). The complement of the soft semi-preopen set, soft regular open set, soft α-open set, soft semi-preopen set, and soft preopen set, are their respective soft semi-preclosed set, soft regular closed set, soft α-closed set, soft semi-closed set, and soft preclosed set.

Definition 1.9:

A soft subset (F, A) in a soft topological space (X, E, τ) is called

(1) a soft generalized closed (briefly soft -closed) in X , if $(F, A) \tilde{\subseteq} (G, B)$ whenever $(F, A) \tilde{\subseteq} (G, B)$ and (G, B) is soft open in X .

(2) a soft semi open, if $(F, A) \tilde{\subseteq}$ $(F, A))$

(3) a soft regular open, if $(F, A) =$ $(F, A))$.

(4) a soft α-open, if $(F, A) \tilde{\subseteq}$ $(F, A)))$

(5) a soft semi preopen or (soft -open), if $(F, A) \tilde{\subseteq}$ $(F, A)))$

(6) a soft pre-open set , if $(F, A) \tilde{\subseteq}$ $(F, A))$.

(7) a soft pre generalized closed (briefly soft -closed) in a soft topological space (X , τ , E) if $(F, A) \tilde{\subseteq} (G, B)$ whenever $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\in}$ SPO(X , τ).

(8) a soft regular generalized closed (briefly soft -closed) in a soft topological space (X , τ , E) if $(F, A) \tilde{\subseteq} (G, B)$ whenever $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\in}$ SRO(X , τ).

Remark 1.10: The Cardinality of $SS(U_A)$ is given by $n(SS(U_A)) = 2^{n(U)\times n(A)}$. That means, if U= { c_1 , c_2 , c_3 , c_4 }and $A = \{e_1, e_2\}$, then $n(SS(U_A)) = 2^{4\times 2} = 256$.

Lemma 1.11: Let (F, A) be a soft set in a soft topological space. Then,

(i) If (F, A) is soft regular open, then (F, A) is soft open.

(ii) If (F, A) is soft open, then (F, A) is soft -open.

(iii) If (F, A) is soft -open , then (F, A) is both soft semi-open and soft pre-open.

(iv) If (F, A) is soft semi-open set, then (F, A) is soft -open.

(v) If (F, A) is soft pre-open set, then (F, A) is soft -open.

Definition:1.12

The finite union of soft regular open sets is called soft π -open set and its complement is soft π -closed set.

Definition:1.13

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Let X be a soft topological space, then X is said to be a soft $T_{1/2}$ -space if any soft g-closed set is soft closed in X .

2. The Notion of Soft $b-$ Open Sets

In this section, we first present some basic definitions and notations on soft $b-$ open sets and several properties of them.

Definition 2.1 A soft set (F,A) in a soft topological space X is called

(a) soft $b-$ open ($sb-$ open) set iff $(F,A) \tilde{\subseteq}$ $(F,A)) \sqcup$ $(F,A))$

(b) soft $b-$ closed ($sb-$ closed) set iff $(F,A)) \sqcap$ $(F,A)) \tilde{\subseteq}$ (F,A). The family of all soft $b-$ open (soft $b-$ closed) sets in a soft topological space (X,E,τ) is denoted by $SBO(X)$ (resp. $SBC(X)$).

Theorem 2.2 For a soft set (F,A) in a soft topological space X

(a) (F,A) is a soft $b-$ open set iff $(F,A)^c$ is a soft $b-$ closed set.

(b) (F,A) is a soft $b-$ closed set iff $(F,A)^c$ is a soft $b-$ open set.

Proof: Obvious from the definition 2.1

Definition 2.3 Let (X,E,τ) be a soft topological space and (F,A) be a soft set over X .

(i) Soft $b-$ closure of a soft set (F,A) in X is denoted by

$$(F,A) = \sqcap\{ (G,B) \setminus (F,A) \tilde{\subseteq} (G,B) : (G,B) \text{ is a soft } b-\text{closed set of } X \}.$$

(ii) Soft $b-$ interior of a soft set (F,A) in X is denoted by

$$(F,A) = \sqcup\{ (G,B) \setminus (G,B) \tilde{\subseteq} (F,A) : (G,B) \text{ is a soft } b-\text{open set of } X \}.$$

Clearly (F,A) is the smallest soft $b-$ closed set over X which contains (F,A) and (F,A) is the largest soft $b-$ open set over X which is contained in (F,A).

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Theorem 2.4 Assume that (F, A) is a soft set in a soft topological space (X, E, τ). Then we consider the following:

(i) (F, A)

(ii) $=$ (F, A)

Proof (i) Let $\{(H_j, D_j)\}_{j\in J}$ be a collection of all the soft $b-$closed sets in X. Then,

$(F, A) = \coprod\{\ (H_j, D_j)^c \mid (F, A)^c\ \tilde{\subseteq} (H_j, D_j) : (H_j, D_j)$ is a soft $b-$closed set of $X\} =$ $\prod\{\ (H_j, D_j) \mid (F, A)^c\ \tilde{\subseteq} (H_j, D_j) : (H_j, D_j)$ is a soft $b-$closed set of $X\} =$. Therefore, $=$ (F, A).

(ii) Let $\{(G_i, B_i)\}_{i\in I}$ be a collection of all the soft $b-$open sets in X .Then,

$(F, A) = \prod\{\ (G_i, B_i)^c \mid (G_i, B_i)\ \tilde{\subseteq} (F, A)^c : (G_i, B_i)$ is a soft $b-$open set of $X\}$ $=$ $\coprod\{\ (G_i, B_i) \mid (G_i, B_i)\ \tilde{\subseteq} (F, A)^c : (G_i, B_i)$ is a soft $b-$ open set of $X\} =$ $(F, A)^c$ Therefore, $=$ (F, A).

Lemma 2.5 Let (F, A) be a soft set in a soft topological space X. Then,

(i) (F, A) = $(F, A) \coprod$ $(F, A))$ and (F, A) = $(F, A) \prod$ $(F, A))$,

(ii) $(F, A) =$ $(F, A) \coprod$ $(F, A))$ and (F, A) $= (F, A) \prod$ $(F, A))$.

Proof (i) $(F, A))\ \tilde{\subseteq}$ $(scl^S(F, A)))\ \tilde{\subseteq}$ (F, A)

$(F, A) \coprod$ $(F, A))\ \tilde{\subseteq} (F, A) \coprod$ $(F, A) =$ (F, A).

So $(F, A) \coprod$ $(F, A))\ \tilde{\subseteq}$ (F, A) …….(1)

Also, its easy from the definition of the soft semi closed set we can show that

$(F, A) \tilde{\subseteq} (F, A) \amalg \operatorname{int}^{S}(cl^{S}(F, A))$ ....(2)

Hence from 1 and 2, $(F, A) = (F, A) \amalg \quad (F, A))$.

(ii) Similar by taking the complements.

Lemma 2.6 In a soft topological space we have the following

(i) Every soft regular open set is soft open.

(ii) Every soft open set is soft α -open.

(iii) Every soft α -open set is both soft semi-open and soft pre-open.

(iv) Every soft semi-open set and every soft pre-open set is soft β -open.

Proof: By using definition 1.8 we have (i), (ii), (iii), and (iv) are hold.

Theorem 2.7 In a soft topological space X

(i) Every soft p-open set is soft $b-$open set.

(ii) Every soft semi-open set is soft $b-$open set.

Proof : (i) Let (F, A) be a soft p-open set in a soft topological space X. Then, (F, A) $\tilde{\subseteq} \quad (F, A))$ which implies $(F, A) \tilde{\subseteq} \quad (F, A)) \amalg$ $(F, A) \tilde{\subseteq} \quad (F, A)) \amalg \quad (F, A))$. Thus (F, A) is soft $b-$open set.

(ii) Let (F, A) be a soft semi-open set in a soft topological space X. Then, $(F, A) \tilde{\subseteq}$ $(F, A))$ which implies $(F, A) \tilde{\subseteq} \quad (F, A)) \amalg \quad (F, A) \tilde{\subseteq}$ $(F, A)) \amalg \quad (F, A))$. Thus (F, A) is soft $b-$open set.

The converses are not true as seen in the following example:

Example 2.8 Let $X = \{a_1,\ a_2,\ a_3, a_4\}$, $E = \{e_1,\ e_2\ ,\ e_3\}$ and

$\tau = \{\Phi, (X,E), (F_1,E), (F_2,E), (F_3,E), ..., (F_{15},E)\}$, where

$(F_1,E), (F_2,E), (F_3,E), ..., (F_{15},E)$ are soft sets over X,

defined as follows:

$(F_1,E) = \{(e_1,\{a_1\}), (e_2,\{a_2,a_3\}), (e_3,\{a_1,a_4\})\}$,

$(F_2,E) = \{(e_1,\{a_2,a_4\}), (e_2,\{a_1,a_3,a_4\}), (e_3,\{a_1,a_2,a_4\})\}$,

$(F_3,E) = \{(e_1,\phi), (e_2,\{a_3\}), (e_3,\{a_1\})\} = \{(e_2,\{a_3\}), (e_3,\{a_1\})\}$,

$(F_4,E) = \{(e_1,\{a_1,a_2,a_4\}), (e_2,X), (e_3,X)\}$,

$(F_5,E) = \{(e_1,\{a_1,a_3\}), (e_2,\{a_2,a_4\}), (e_3,\{a_2\})\}$,

$(F_6,E) = \{(e_1,\{a_1\}), (e_2,\{a_2\})\}$,

$(F_7,E) = \{(e_1,\{a_1,a_3\}), (e_2,\{a_2,a_3,a_4\}), (e_3,\{a_1,a_2,a_4\})\}$,

$(F_8,E) = \{(e_2,\{a_4\}), (e_3,\{a_2\})\}$,

$(F_9,E) = \{(e_1,X), (e_2,X), (e_3,\{a_1,a_2,a_4\})\}$,

$(F_{10},E) = \{(e_1,\{a_1,a_3\}), (e_2,\{a_2,a_3,a_4\}), (e_3,\{a_1,a_2\})\}$,

$(F_{11},E) = \{(e_1,\{a_1,a_2,a_4\}), (e_2,X), (e_3,\{a_1,a_2,a_4\})\}$,

$(F_{12},E) = \{(e_1,\{a_1\}), (e_2,\{a_2,a_3,a_4\}), (e_3,\{a_1,a_2,a_4\})\}$,

$(F_{13},E) = \{(e_1,\{a_1\}), (e_2,\{a_2,a_4\}), (e_3,\{a_2\})\}$,

$(F_{14},E) = \{(e_1,\{a_1\}), (e_2,\{a_2,a_3,a_4\}), (e_3,\{a_1,a_2\})\}$,

$(F_{15},E) = \{(e_1,\{a_1\}), (e_2,\{a_2,a_3\}), (e_3,\{a_1\})\}$.

Then, τ defines a soft topology on X, and thus (X,E,τ) is a soft topological space over X. Clearly, the soft closed sets are

$\Phi, (X,E), (F_1,E)^c, (F_2,E)^c, (F_3,E)^c, ..., (F_{15},E)^c$ Then, let us

$(F,A) = \{(e_1,\{a_2,a_4\}), (e_2,\{a_1,a_3\}), (e_3,\{a_1,a_3,a_4\})\}$; therefore $(F,A))$ ∐

$(F,A)) = X$, and so $(F,A) \tilde{\subseteq}$ $(F,A)) \coprod$ $(F,A))$; hence, (F,A) is soft $b-$open set, but not soft p-open set (since (F,A) is not soft p-open set). Now, let us take $(G,E) = \{(e_1,\{a_4\}),(e_2,\{a_1,a_2,a_3\}),(e_3,\{a_2,a_4\})\}$; then (G,E) $\coprod$ $(G,E)) = X$, and so $(G,E) \tilde{\subseteq}$ $(F,A)) \coprod$ (G,E) ; is soft $b-$open set, but not soft semi-open set.

Remark 2.9

(i) If (F,A) is a soft set of soft topological space X , then $bcl^S(F,A)$ is the smallest soft $b-$closed set containing (F,A). Thus,

$bcl^S(F,A) = (F,A) \coprod [cl^S(\text{int}^S(F,A)) \prod \text{int}^S(cl^S(F,A))]$.

(ii) If (F,A) is a soft set of soft topological space X , then $b\,\text{int}^s(F,A)$ is the biggest soft $b-$open set contained (F,A). Thus,

$b\,\text{int}^S(F,A) = (F,A) \prod [cl^S(\text{int}^S(F,A)) \coprod \text{int}^S(cl^S(F,A))]$.

Theorem 2.10 In a soft topological space X , every soft $b-$open (soft $b-$closed) set is soft β -open (soft β -closed) set.

Proof Let (F,A) be a soft $b-$open set in X . Then we get (F,A) $\tilde{\subseteq} cl^S(\text{int}^S(F,A)) \coprod \text{int}^S(cl^S(F,A)) \tilde{\subseteq} cl^S(\text{int}^S(cl^S(F,A))) \coprod \text{int}^S(cl^S(F,A)) \tilde{\subseteq}$ $cl^S(\text{int}^S(cl^S(F,A)))$. As a result (F,A) is soft β -open set.

Note. The converse is not true as seen in the following example:

Example 2.11

Let Let $X = \{a_1, a_2, a_3, a_4\}$, $E = \{e_1, e_2, e_3\}$ and let (X,E,τ) be soft topological space over X . Let us consider the soft topology τ on X given in

Example 2.8; i.e., $\tau = \{\Phi,(X,E),(F_1,E),(F_2,E),(F_3,E),...,(F_{15},E)\}$. Then, let us $(H,E)=\{(e_1,\{a_1,a_2\}),(e_2,\{a_3,a_4\}),(e_3,\{a_1,a_3,a_4\})\}$. Then, $cl^S(\text{int}^S(cl^S(H,E)))$ $=\{(e_1,\{a_2,a_3,a_4\}),(e_2,\{a_1,a_3,a_4\}),(e_3,X)\}$, and so $(H,E)\tilde{\subseteq} cl^S(\text{int}^S(cl^S(H,E)))$, therefore (H,E) is soft β-open set but not soft $b-$open set.

Remark 2.12 From the above theorems, we have the following diagram

soft regular set $\Rightarrow$ soft open set

$\Downarrow$

α-open set soft

$\Downarrow$ $\Downarrow$

soft semi -open set soft pre-open set

$\Downarrow$ $\Downarrow$

soft $b-$open set

$\Downarrow$

soft β-open set

2.13 Soft $b-$continuity

In this subsection, we introduce soft $b-$continuous maps, soft $b-$irresolute maps, soft $b-$closed maps and soft $b-$open maps and study some of their properties.

Definition 2.13.1 Let (X,E) and (Y,K) be soft classes and let $u: X \to Y$ and $p: E \to K$ be mappings. Then a mapping $f: (X,E) \to (Y,K)$ is defined as: for a soft set (F,A) in (X,E), $(f(F,A),B)$, $B=p(A)\subseteq K$ is a soft set in (Y,K) given by

$f(F,A)(\beta)=u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)\right)$ for $\beta\in K$. $(f(F,A),B)$ is called a soft image of a soft set (F,A). If B = K, then we shall write $(f(F,A),K)$ as $f(F,A)$.

Definition 2.13.2 Let $f:\ (X,E)\to(Y,K)$ be a mapping from a soft class (X,E) to another soft class (Y,K) and (G,C) a soft set in soft class (Y,K) where $C\subseteq K$. Let $u:X\to Y$ and $p:E\to K$ be mappings. Then $(f^{-1}(G,C),D)$, $D=p^{-1}(C)$, is a soft set in the soft classes (X,E) defined as: $f^{-1}(G,C)(\alpha)=u^{-1}(G(p(\alpha)))$ for $\alpha\in D\subseteq E$. $(f^{-1}(G,C),D)$, is called a soft inverse image of (G,C). Hereafter, we shall write $(f^{-1}(G,C),E)$ as $f^{-1}(G,C)$.

Theorem 2.13.3 Let $f:(X,E)\to(Y,K)$, $u:X\to Y$ and $p:E\to K$ be mappings. Then for soft sets (F,A), (G,B) and a family of soft sets (F_i,A_i) in the soft class (X,E) we have:

(1) $f(\Phi)=\Phi$,

(2) $f((X,E))=(Y,K)$,

(3) $f((F,A)\coprod(G,B))=f((F,A))\coprod f((G,B))$ in general $f(\coprod_{i\in I}(F_i,A_i))=\coprod_{i\in I} f((F_i,A_i))$,

(4) $f((F,A)\prod(G,B))\subseteq f((F,A))\prod f((G,B))$ in general $f(\prod_{i\in I}(F_i,A_i))\subseteq\prod_{i\in I} f((F_i,A_i))$,

(5) If $(F,A)\subseteq(G,B)$ then $f((F,A))\subseteq f((G,B))$,

(6) $f^{-1}(\Phi)=\Phi$,

(7) $f^{-1}((Y,K))=(X,E)$,

(8) $f^{-1}((F,A)\coprod(G,B))=f^{-1}((F,A))\coprod f^{-1}((G,B))$ in general $f^{-1}(\coprod_{i\in I}(F_i,A_i))=\coprod_{i\in I} f^{-1}((F_i,A_i))$,

(9) $f^{-1}((F,A)\prod(G,B)) = f^{-1}((F,A))\prod f^{-1}((G,B))$ in general

$f^{-1}(\prod_{i\in I}(F_i,A_i)) = \prod_{i\in I} f^{-1}((F_i,A_i))$,

(10) If $(F,A)\subseteq(G,B)$ then $f^{-1}((F,A))\subseteq f^{-1}((G,B))$.

Proof : The proof is obvious.

Definition 2.13.4 A soft mapping $f:(X,E)\to(Y,K)$ is said to be soft $b-$ continuous (briefly sb -continuous) if the inverse image of each soft open set of (Y,K) is a s$b-$open set in (X,E).

Definition 2.13.5 A soft mapping $f:(X,E)\to(Y,K)$ is called soft continuous (resp., soft α - continuous, soft pre-continuous, soft semi-continuous, soft β -continuous) if the inverse image of each soft open set in Y is soft open (resp. s α -open, sp-open, ss-open, s β -open) set in X .

Theorem 2.13.6 Let $f:(X,E)\to(Y,K)$ be a mapping from a soft space X to soft space Y. Then, the following statements are true;

(i) f is s$b-$continuous,

(ii) the inverse image of each soft closed set in Y is soft $b-$closed in X .

Proof (i) $\Rightarrow$ (ii): Let (G,K) be a soft closed set in Y. Then $(G,K)^c$ is soft open set. Thus, $f^{-1}((G,K)^c)\in SBO(X)$ i.e., $(X,E)-f^{-1}((G,K))\in SBO(X)$. Hence $f^{-1}((G,K))$ is a s$b-$closed set in X .

(ii) $\Rightarrow$ (i): Let (O,K) is soft open set in Y. Then $(O,K)^c$ is soft closed set and by (ii) we have $(X,E)-f^{-1}((O,K)^c)\in SBO(X)$, i.e., $(X,E)-f^{-1}((O,K))\in SBO(X)$. Hence $f^{-1}((O,K))$ is a s$b-$open set in X .Therefore, f is a s$b-$continuous function.

Remark 2.13.7 We have following implications, however, examples given below show that the converses of these implications are not true.

soft continuity

$\Downarrow$

α - continuity soft

$\Downarrow$ $\qquad$ $\Downarrow$

soft semi - continuity $\qquad$ soft pre- continuity

$\Downarrow$ $\qquad$ $\Downarrow$

soft $b-$ continuity

$\Downarrow$

soft β - continuity

Example 2.13.8 Let $X=\{a_1, a_2, a_3, a_4\}$, $Y=\{y_1, y_2, y_3, y_4\}$, $E=\{e_1, e_2, e_3\}$ and $K=\{k_1, k_2, k_3\}$ and let (X,E,τ) and (Y,K,υ) be soft topological spaces.

Define $u: X \to Y$ and $p: E \to K$ as

$u(a_1)=\{y_1\}, u(a_2)=\{y_3\}, u(a_3)=\{y_2\}, u(a_4)=\{y_4\}$,

$p(e_1)=\{k_2\}, p(e_2)=\{k_1\}, p(e_3)=\{k_3\}$,

Let us consider the soft topology τ on X given in Example 2.8; i.e.,

$\tau = \{\Phi, (X,E), (F_1,E), (F_2,E), (F_3,E), ..., (F_{15},E)\}$,

$\upsilon = \{\Phi, (Y,K), (L,K)\}$,

$(L,K) = \{(k_1,\{y_2,y_4\}), (k_2,\{y_1,y_3\}), (k_3,\{y_1,y_2,y_4\})\}$

and mapping; $f: (X,E,\tau) \to (Y,K,\upsilon)$ is a soft mapping. Then (L,K) is a soft open set in Y, $f^{-1}((L,K)) = \{(e_1,\{a_1,a_2\}), (e_2,\{a_3,a_4\}), (e_3,\{a_1,a_3,a_4\})\}$ is a $s\beta$-open set but not $sb-$open set in X. Therefore, f is a soft β-continuous function but not $sb-$continuous function.

Example 2.13.9 Let $X=\{a_1, a_2, a_3, a_4\}$, $Y=\{y_1, y_2, y_3, y_4\}$, $E=\{e_1, e_2, e_3\}$ and $K=\{k_1, k_2, k_3\}$ and let (X,E,τ) and (Y,K,υ) be soft topological spaces.

Define $u: X \to Y$ and $p: E \to K$ as

$u(a_1)=\{y_1\}, u(a_2)=\{y_3\}, u(a_3)=\{y_2\}, u(a_4)=\{y_4\}$,

$p(e_1)=\{k_2\}, p(e_2)=\{k_1\}, p(e_3)=\{k_3\}$,

Let us consider the soft topology τ on X given in Example 2.8; i.e.,

$\tau = \{\Phi, (X,E), (F_1,E), (F_2,E), (F_3,E), ..., (F_{15},E)\}$,

$\upsilon = \{\Phi, (Y,K), (M,K)\}$;

$(M,K)= \{(k_1,\{y_1,y_2,y_3\}), (k_2,\{y_4\}), (k_3,\{y_3,y_4\})\}$

and mapping, $f: (X,E,\tau) \to (Y,K,\upsilon)$ is a soft mapping. Then (M,K) is a soft open set in Y, $f^{-1}((M,K))=\{(e_1,\{a_4\}), (e_2,\{a_1,a_2,a_3\}), (e_3,\{a_2,a_4\})\}$ is a s$b-$open set but not ss-open set in X .Hence, f is a s$b-$continuous function but not soft semi-continuous function.

Example 2.13.10 Let $X=\{a_1, a_2, a_3, a_4\}$, $Y=\{y_1, y_2, y_3, y_4\}$, $E=\{e_1, e_2, e_3\}$ and $K=\{k_1, k_2, k_3\}$ and let (X,E,τ) and (Y,K,υ) be soft topological spaces.

Define $u: X \to Y$ and $p: E \to K$ as

$u(a_1)=\{y_1\}, u(a_2)=\{y_3\}, u(a_3)=\{y_2\}, u(a_4)=\{y_4\}$,

$p(e_1)=\{k_2\}, p(e_2)=\{k_1\}, p(e_3)=\{k_3\}$,

Let us consider the soft topology τ on X given in,Example 2.8; i.e.,

$\tau = \{\Phi, (X,E), (F_1,E), (F_2,E), (F_3,E), ..., (F_{15},E)\}$, $\upsilon =$

$\{\Phi, (Y,K), (N,K)\}; (N,K)=\{(k_1,\{y_1,y_2\}), (k_2,\{y_3,y_4\}), (k_3,\{y_1,y_2,y_4\})\}$ and mapping; $f: (X,E,\tau) \to (Y,K,\upsilon)$ is a soft mapping. Then (N,K) is a soft open set in Y, $f^{-1}((N,K))=\{(e_1,\{a_2,a_4\}), (e_2,\{a_1,a_3\}), (e_3,\{a_1,a_3,a_4\})\}$ is a s$b-$open set